Drift-kinetic model of the inner heliosphere

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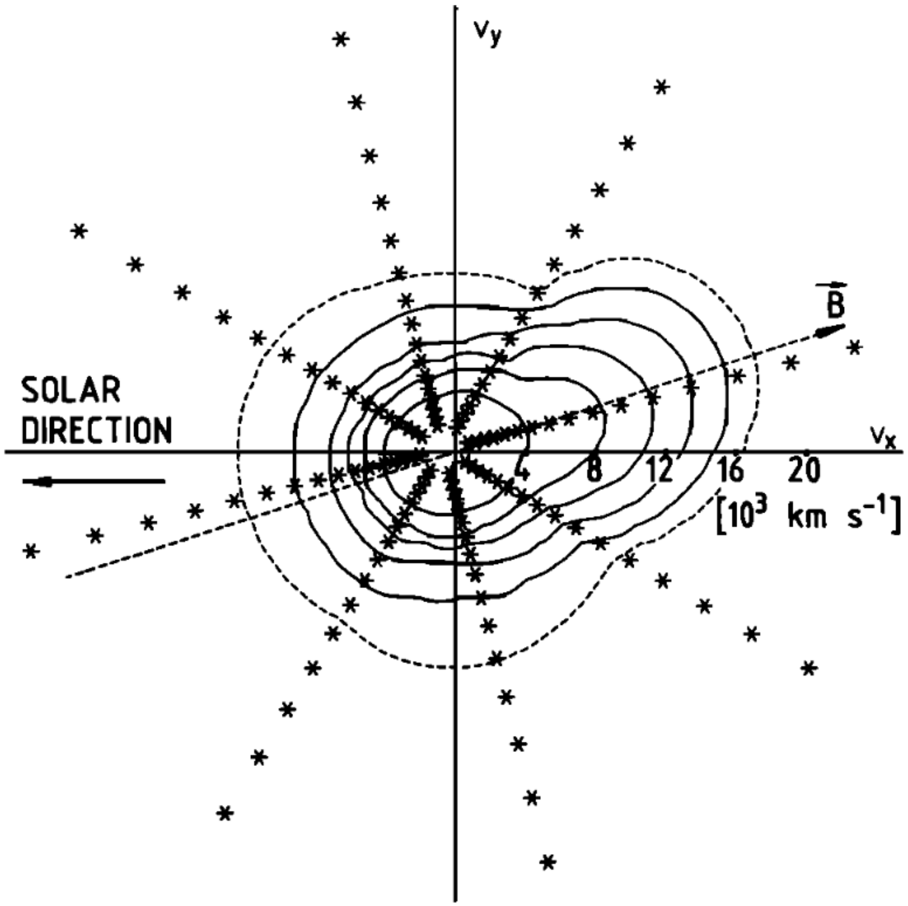
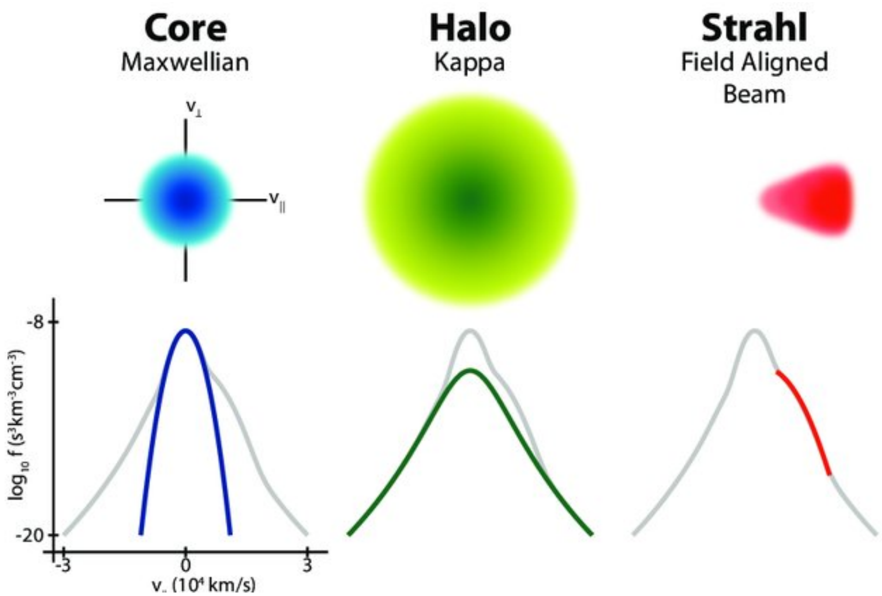
Electron Velocity Distribution Function
at 0.29 AU measured with Helios 2



Credit: Philipp, et al. 1987



Credit: M. Pulupa

The escape velocity is given by

$$v_{esc} = \sqrt{\frac{G M_\odot}{R_\odot}} = \sqrt{2\, g_\odot\, R_\odot} = 6.2 \cdot 10^5\,\mathrm{m/s}$$

compared to a thermal speed of an electron at $T = 4000$ K

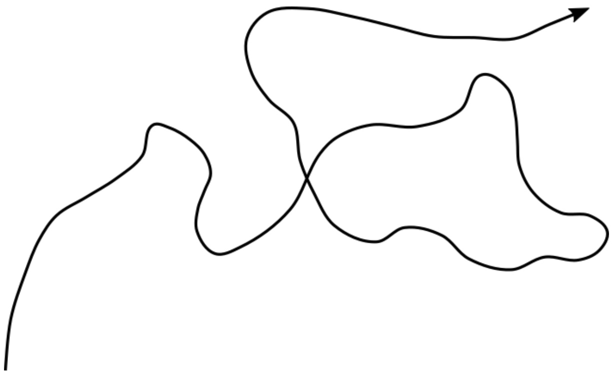
$$v_{th,e} = \sqrt{\frac{k_B T}{m_e}} = 2.6 \cdot 10^5\,\mathrm{m/s}$$

or a thermal speed for a proton

$$v_{th,p} = \sqrt{\frac{k_B T}{m_p}} = 6.0 \cdot 10^3\,\mathrm{m/s}$$

$\Rightarrow$ 5 percent of electrons could boil off, but none of the protons

In a warm dilute plasma collisions are dominated by many successive small angle scatterings



- Plasma is nearly but not exactly neutral
- Fluctuations in the electric field
- Constant small angle scattering
- Mean free path $\lambda_{mfp} = \frac{\sqrt{3\,k_B\,T/m}}{\nu_{e,i}}$
- Effective collision frequency $\nu_{e,i} = \frac{e^4 \log\Lambda}{4\pi\,\epsilon_0^2\,k_B^{3/2}\,m^{1/2}} \frac{n}{T^{3/2}}$
- Actual collision frequency is velocity dependant:

$$\nu_{e,i} = \frac{n\,e^4 \log\Lambda}{4\pi\,\epsilon_0^2\,m^2} \frac{1}{v^3}$$

- Hot dilute plasma like the solar wind has very long mean free path

A steady-state electric field in a plasma will accelerate the charged particles.

Small field

- Particle will collide before it gains a lot of energy
- Finite slip speed between electrons and ions
- Distribution function remains Maxwellian

Large field

- Particle gains more than one thermal energy before the next collision
- Particles experience run-away
- Distribution function become very non-local

The critical value of the electric field that separated the two regimes is the Dreicer electric field

$$E_D = \frac{e}{4\pi\,\epsilon_0\,\lambda_D^2}\log\Lambda$$

Electric Field

We expect an electric field that holds back electron and that ensures

- local quasi-neutrality, despite different scale heights
- no net current, despite tightly bound ions

We can actually estimate the required electric field by including it in the momentum balance for both species and find

$$E_r(r) \approx \frac{m_i\, g_\odot}{e} \frac{R_\odot^2}{r^2} \approx 2\,\mu\mathrm{V}/\mathrm{m} \cdot \frac{R_\odot^2}{r^2}$$

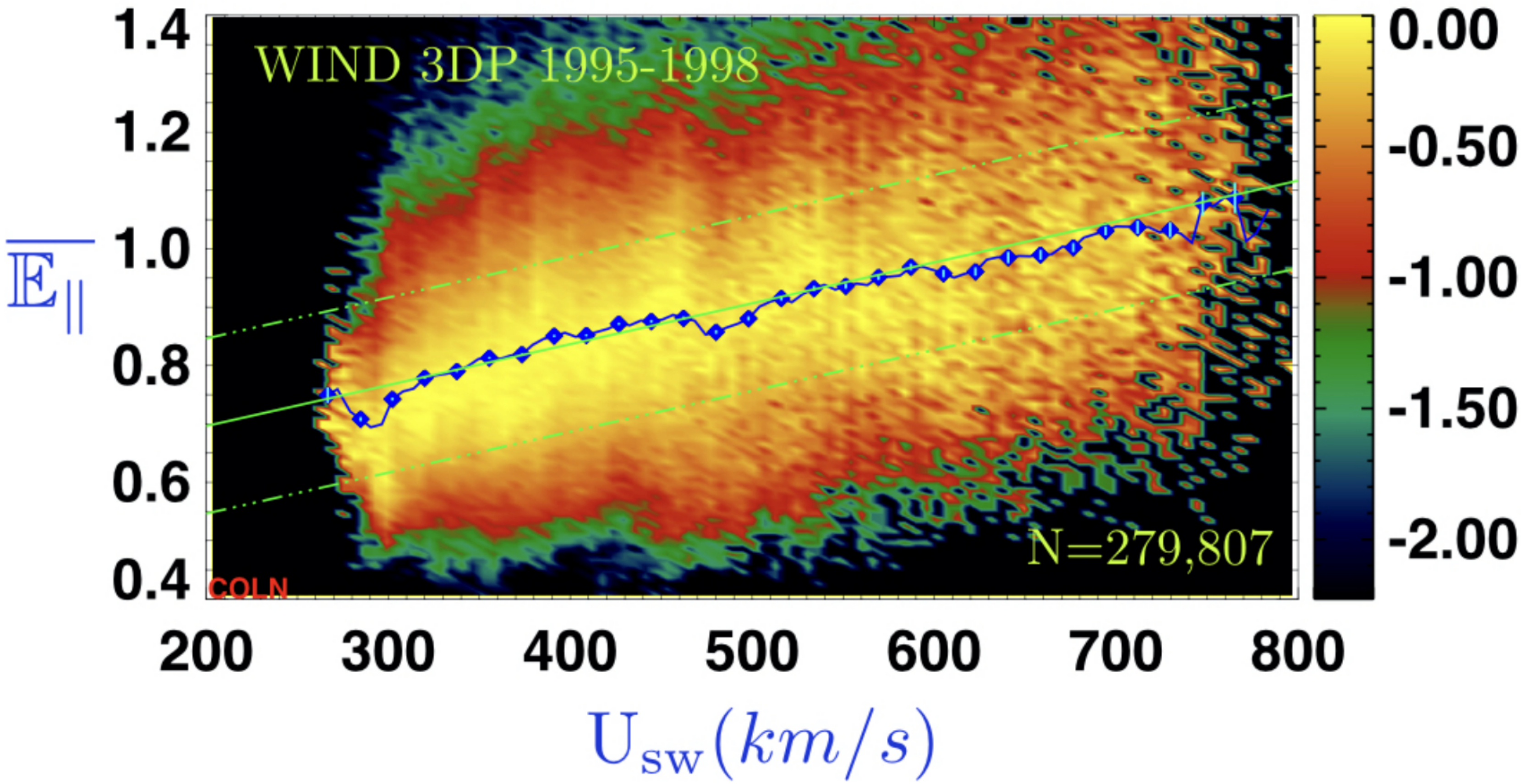
This field sounds small, but is not much smaller than the Dreicer electric field E_D.

The fastest, least collisional electrons can experience run-away.

The slower particles in the core stay essentially Maxwellian.

Yet observations show that the solar wind remains current free.

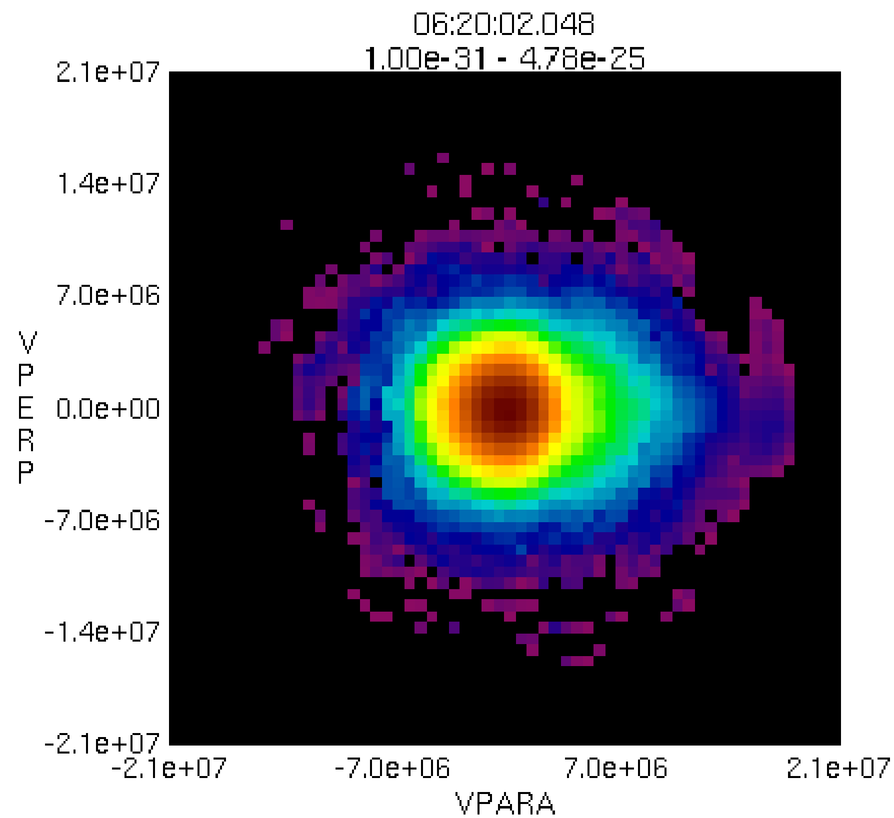
Normalized electric field $\bar{E}_{\parallel} = E_{\parallel}/E_D$ as a function of solar wind speed U_{SW} at 1 AU.



Credit: Jack Scudder

Highly unusual system

- Large scale steady-state parallel electric field
- Sufficient for 10 to 20 percent of particles to run away
- Small enough for a thermal core to remain
- Quasi-neutral plasma
- Zero electrical current but sizable heat flux
- Non-thermal particle distribution without waves or localized acceleration, just from strong gradients.



Credit: Vinas et al, 2010.

Simulations

- + We solve for the distribution functions $f_s(r, v_\parallel)$ or $f_s(r, v_\parallel, v_\perp)$
- + and the self-consistent electric field $E(r)$ that gives charge neutrality.
- o Previous research has used fixed Maxwellian background
  and only solved for fast particles.
- + Next step is to switch on velocity-dependent scattering.
- o Previous research simplified collision operators.
- + We retain the full non-linear collision operator

Solve kinetic equation given a $E(r)$:

$$\overset{\text{steady state}}{\cancel{\frac{\partial f_S}{\partial t}}} + v_R \frac{\partial f_S}{\partial R} + \left(-\frac{G\,M_\odot}{R^2} + \frac{q\,E(r)}{m} + \frac{v_\perp^2}{R} \right) \frac{\partial f}{\partial v_R} - \frac{v_r v_\perp}{R} \frac{\partial f_s}{\partial v_\perp} = \sum_{s'} \mathbb{C}_{s,s'}\{\bar{f}_s, \bar{f}'_s\}$$

compute

$$n_s = \int_0^\infty \int_{-\infty}^\infty 2\pi\, v_\perp\, f_s \,\mathrm{d}v_\parallel \,\mathrm{d}v_\perp$$

find $E(r)$ such that $n_i - n_e = 0$

Collision operator involves Rosenbluth potentials:

$$\sum_{s'} \mathbb{C}_{s,s'}\{\bar{f}_s, \bar{f}'_s\} = \sum_{s'} \frac{2\pi q_s^2 q_{s'}^2 \ln\Lambda}{m_s m_{s'}} \nabla_v \cdot \left(\frac{m_{s'}}{m_s} \nabla\nabla G_{s'} \cdot \nabla f_s - 2 f_s \nabla H_{s'} \right)$$

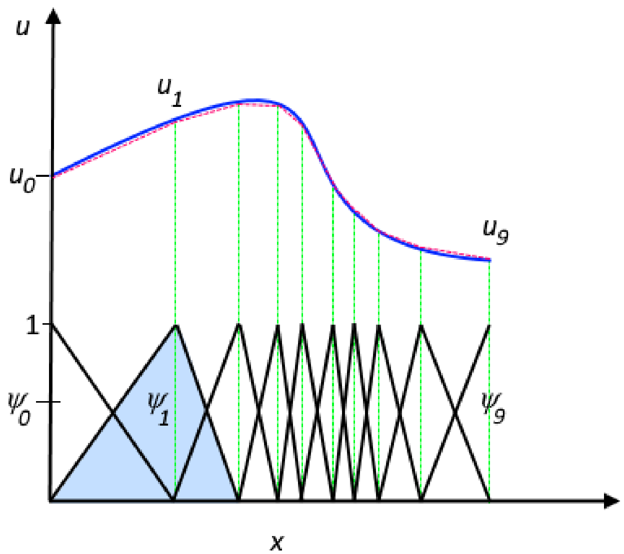
$$\nabla^2 H_{s'} = -2 f_{s'}$$

$$\nabla^2 G_{s'} = 2 H_{s'}$$

This requires Poison equations in velocity space.

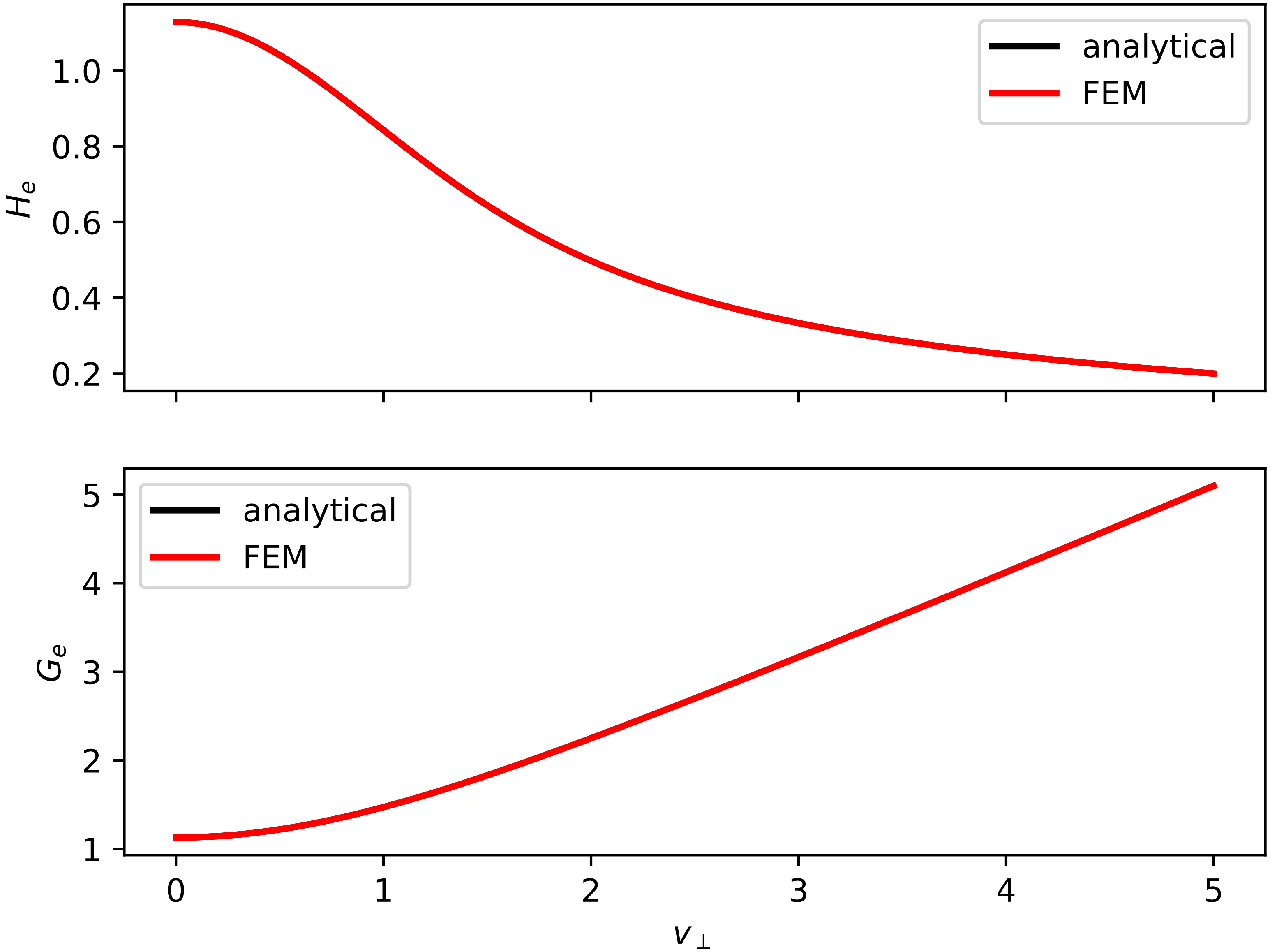
Finite Element Method (FEM)

- Uses a **weak formulation**, i.e. convert into integral problem after multiplication with a test function.
- Describe (unknown) function as **weighted sum of basis functions**.
- Results in **matrix equations** for the (unknown) coefficients.
- Actually uses a **discontinuous Galerkin** method which allows for sharp jump,
  e.g. if parts of phase space are inaccessible.

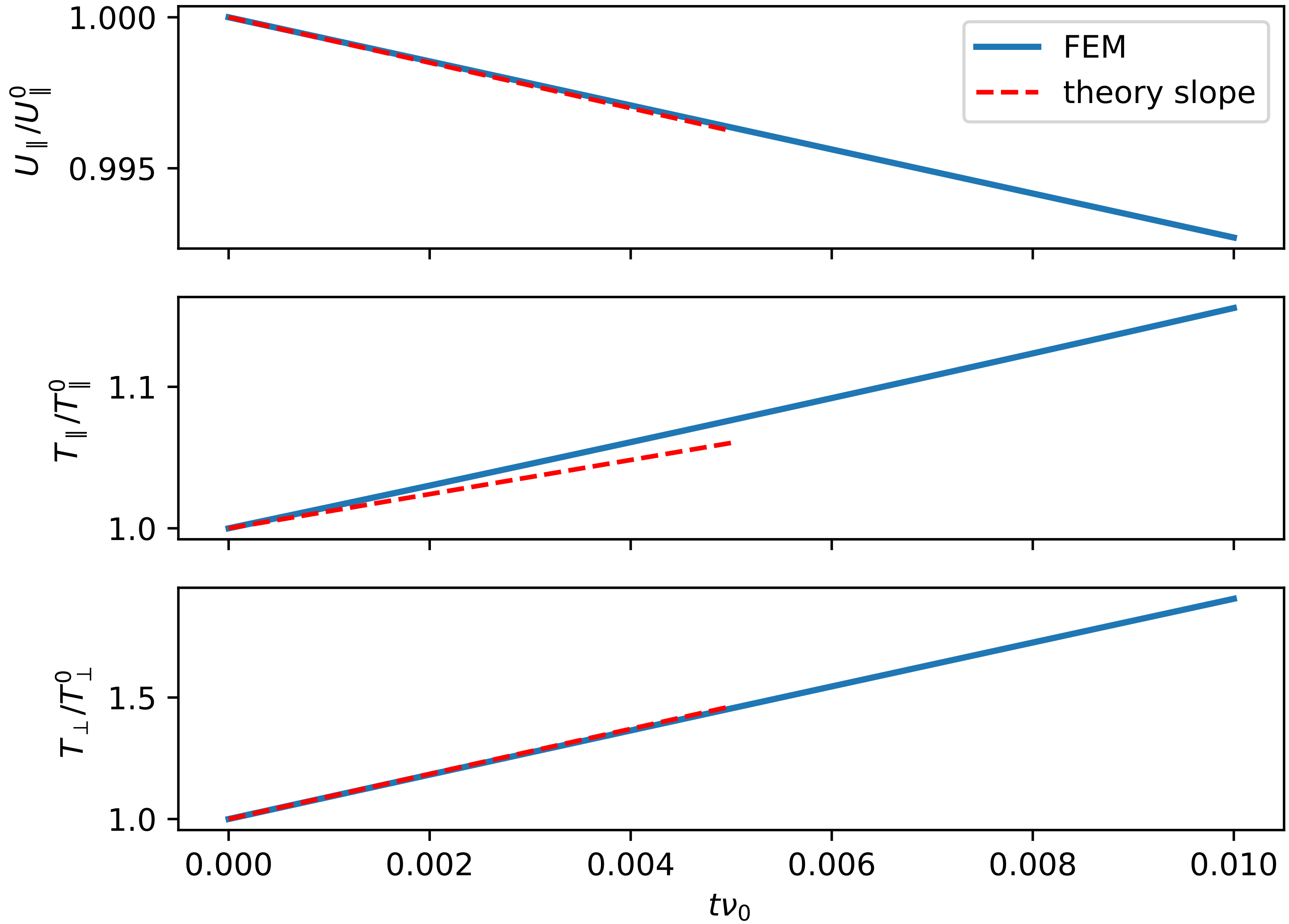


Compared to a spectral expansion in velocity space:

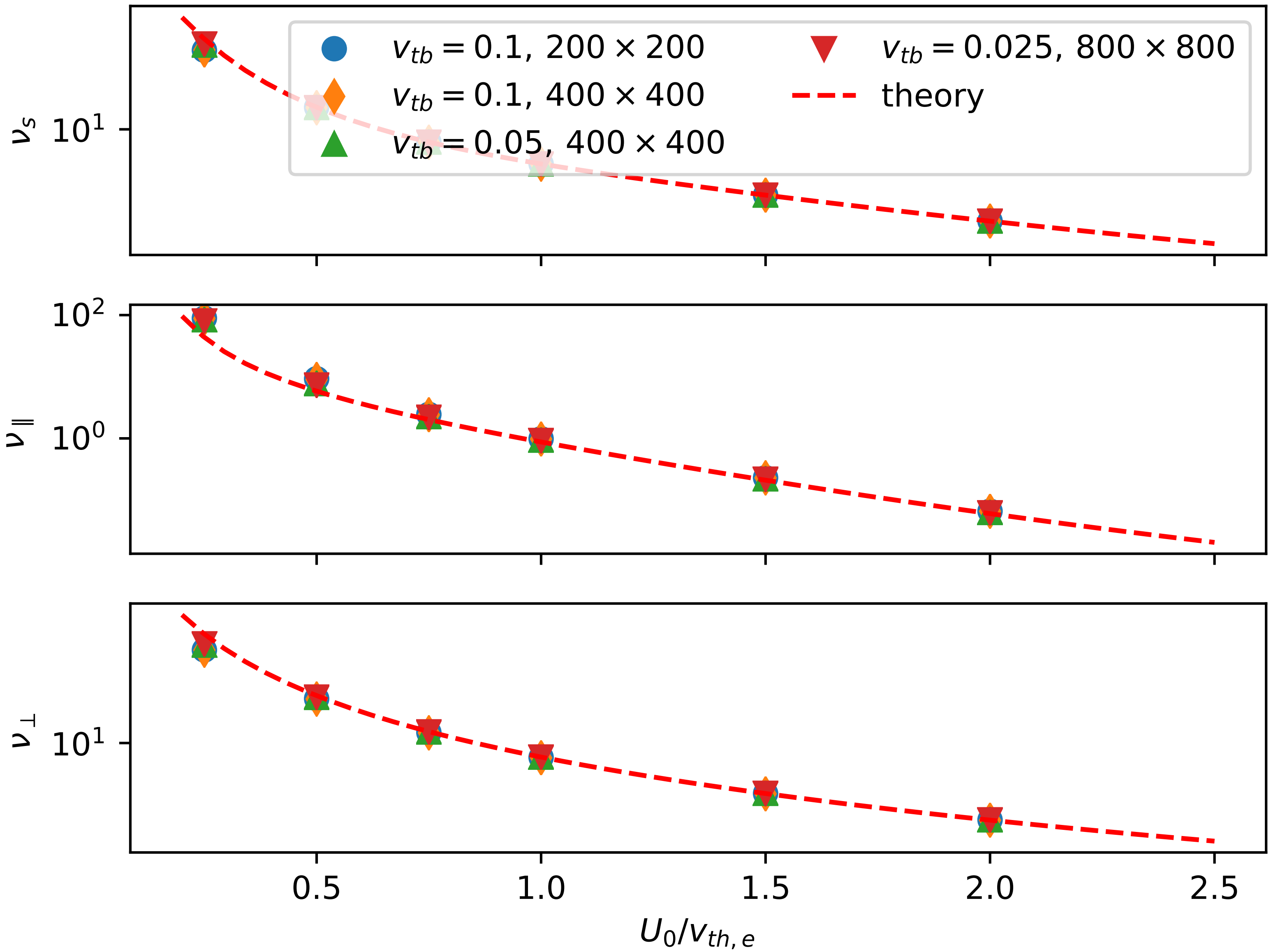
- Wide freedom of the shape of the distribution function
- Several good frameworks available
- Ease of prototyping



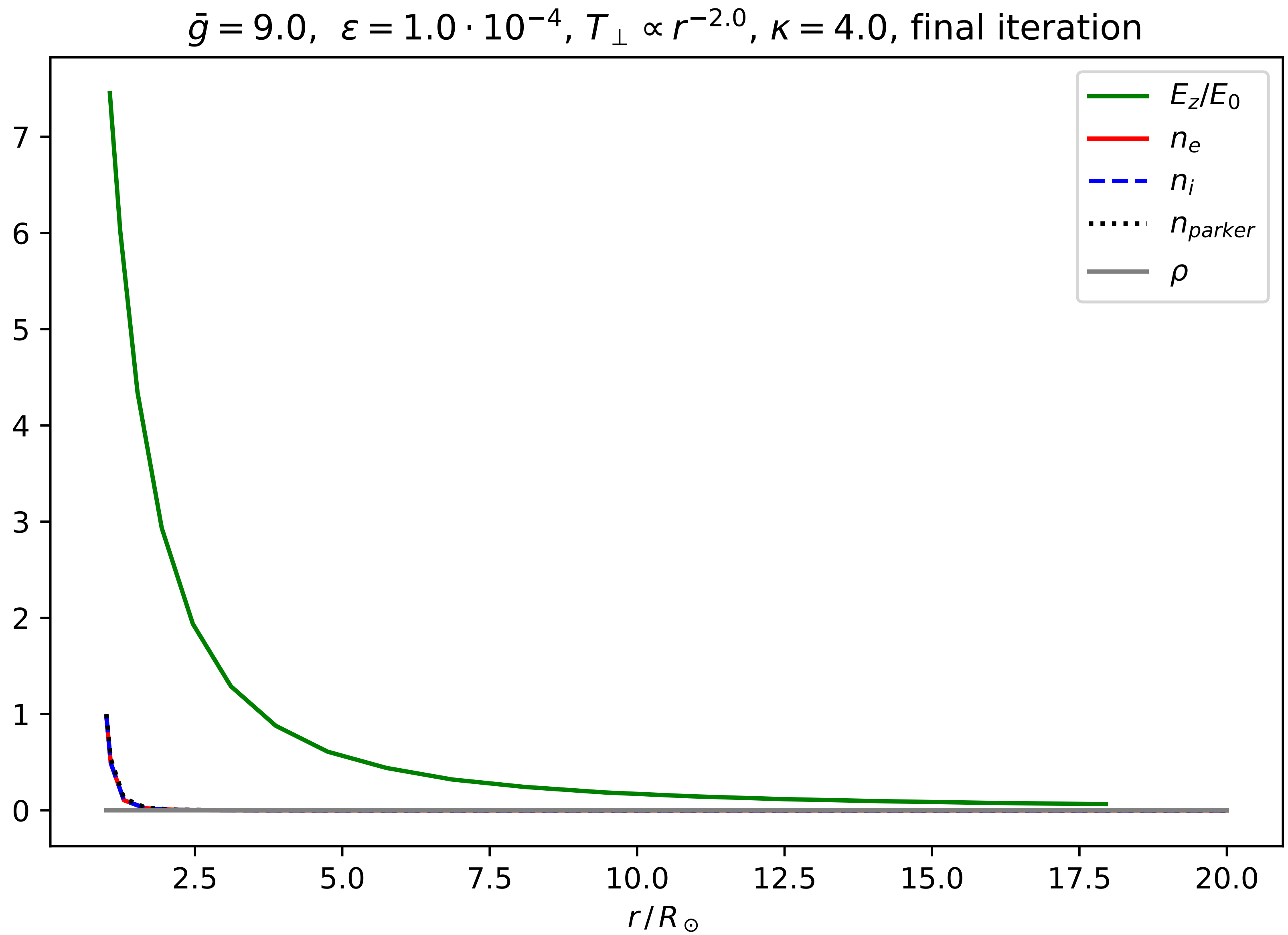
Rosenbluth potentials for a Maxwellian compared to the analytic solution.
Grid up to $v_\perp^{max} = 5v_{th,e}$, $v_\parallel^{max} = 5v_{th,e}$ with 200×200 elements of order 2.



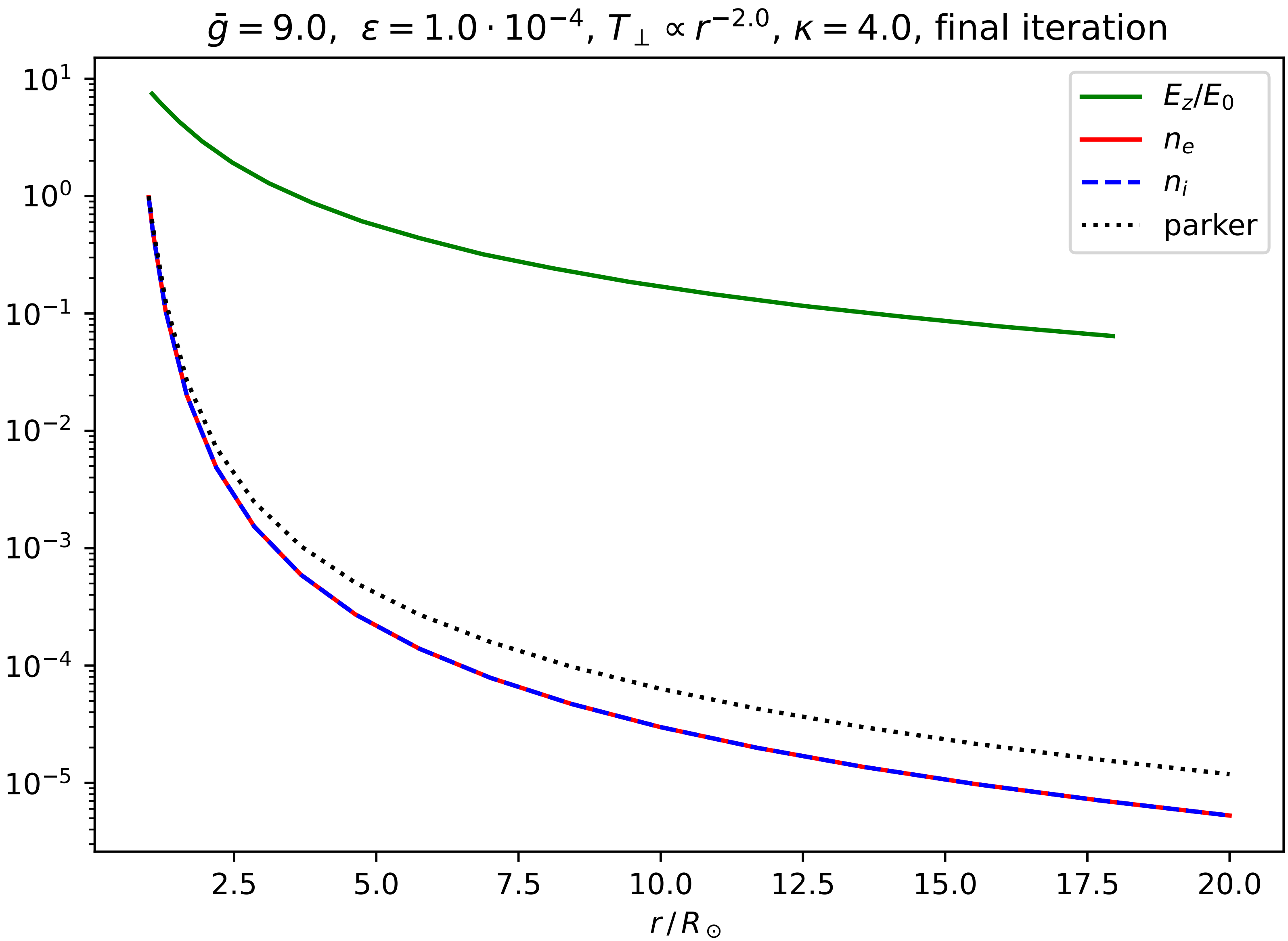
The slow-down (top), parallel diffusion (middle), and perpendicular diffusion (bottom) rates for a test case of electrons scattering from background of Maxwellian electrons and ions. The theoretical slopes are shown by red dashed lines. Background parameters are $m_i/m_e = 100$, $T_i = T_e$, the beam is initially Maxwellian with $U_0 = 2v_{th,e}$ and $T_b = 0.02T_e$.



The slow-down (top), parallel diffusion (middle), and perpendicular diffusion (bottom) rates as a function of beam velocity for a test case of electrons scattering from background of Maxwellian electrons and ions. The theoretical dependencies are shown by red dashed lines.



$\bar{g} = 9.0,\ \varepsilon = 1.0 \cdot 10^{-4}, T_\perp \propto r^{-2.0}, \kappa = 4.0$, final iteration
E_z/E_0
n_e
n_i
n_{parker}
ρ
0
1
2
3
4
5
6
7
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$r\,/\,R_\odot$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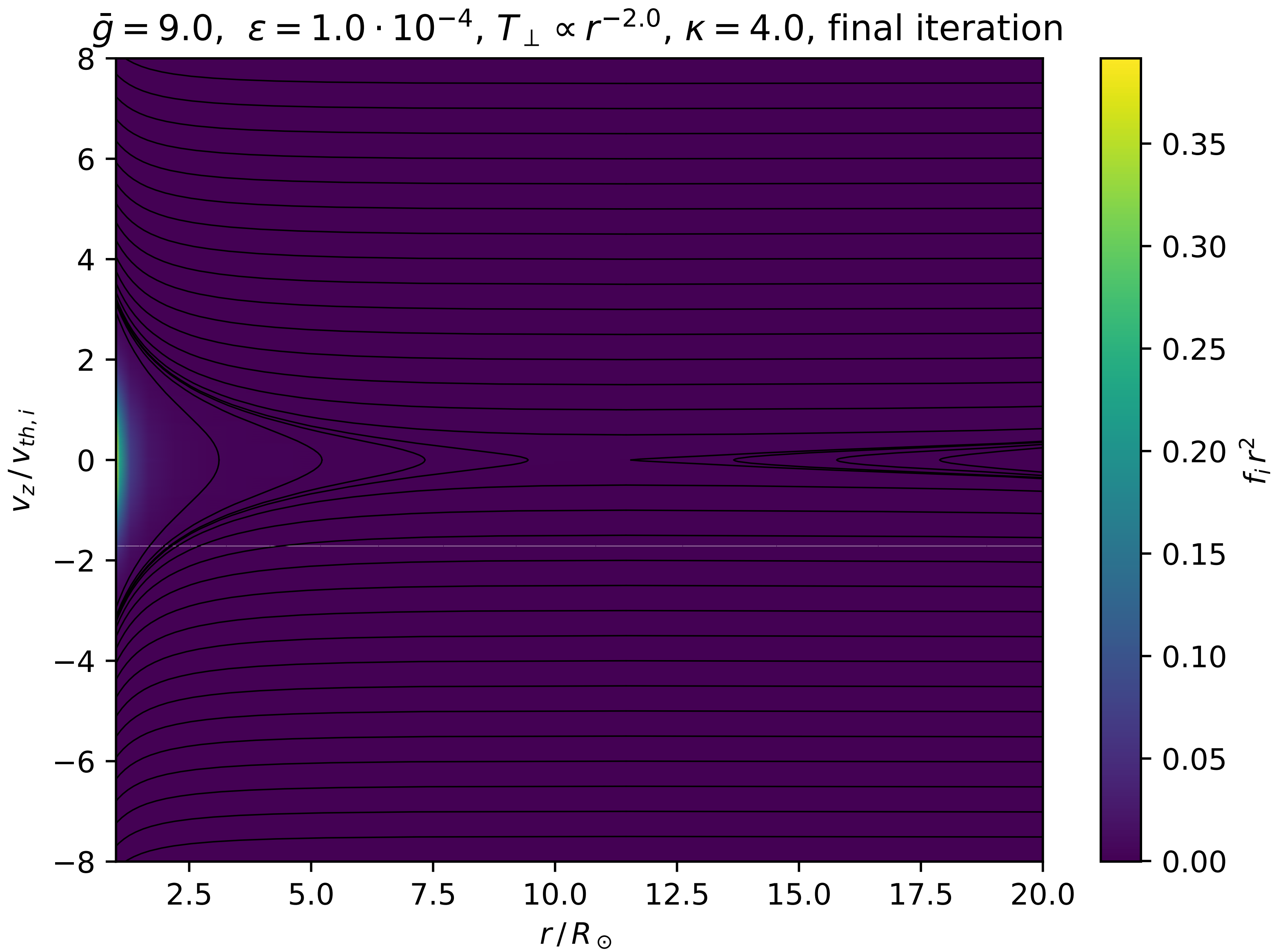
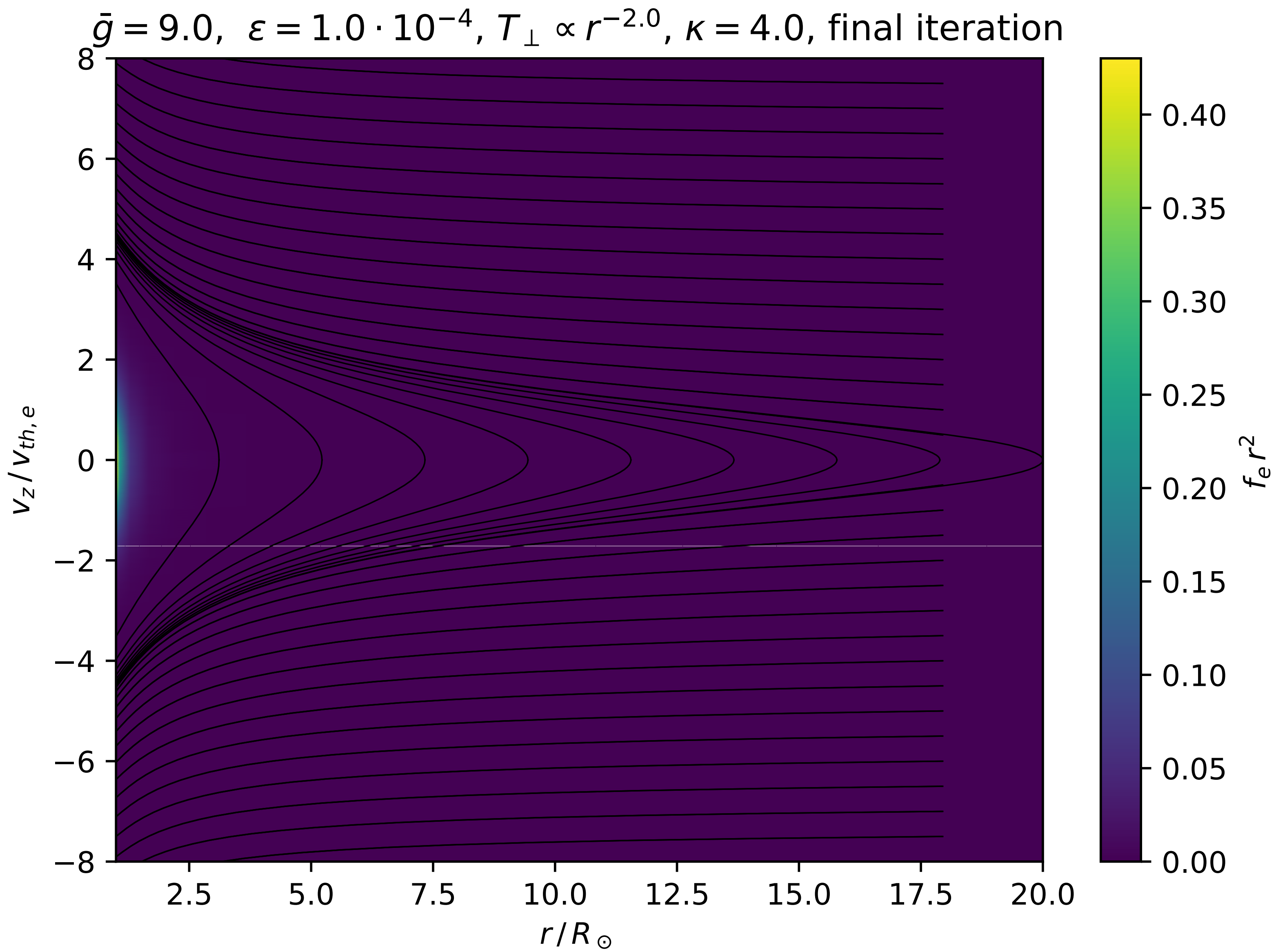
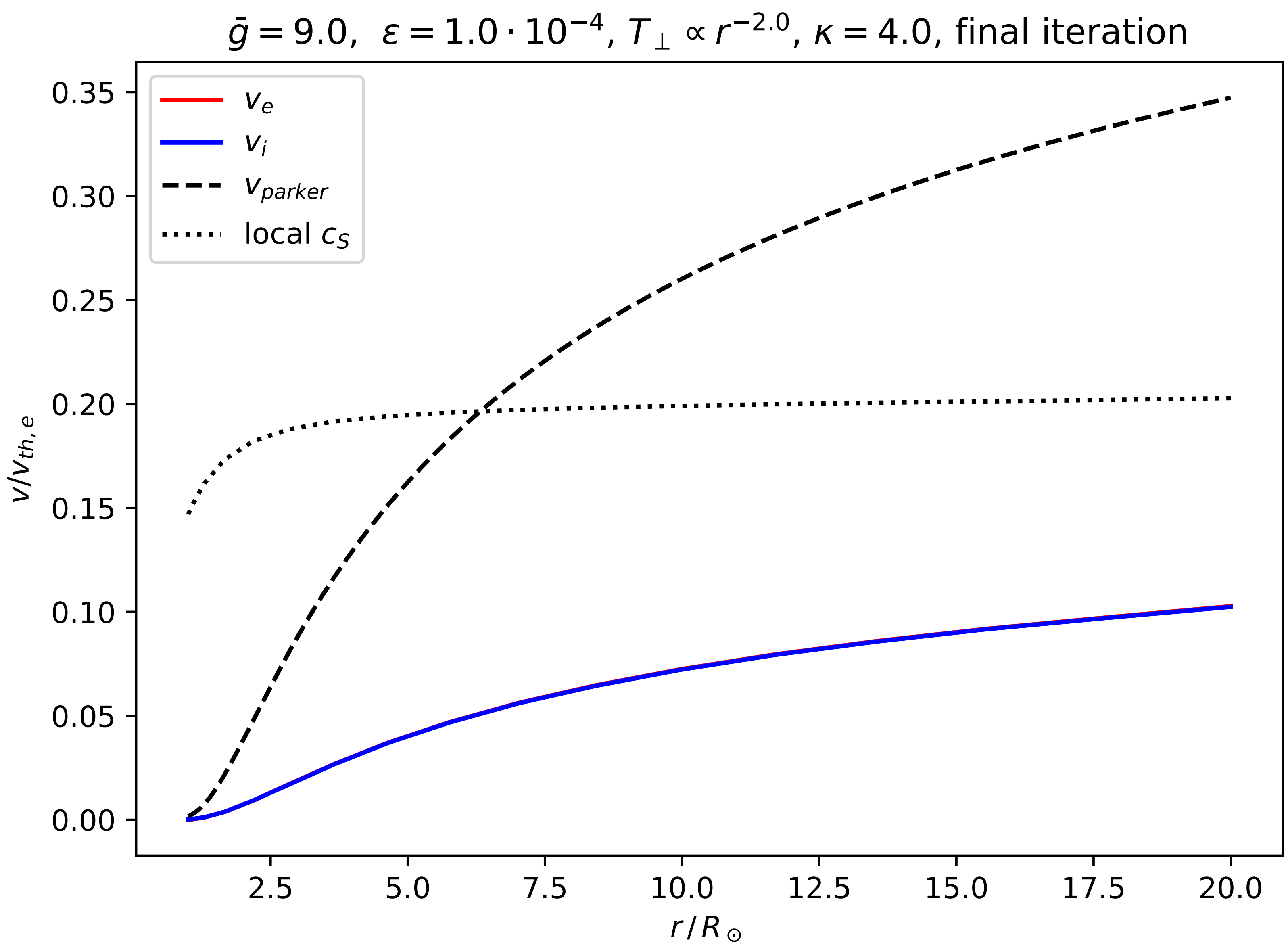
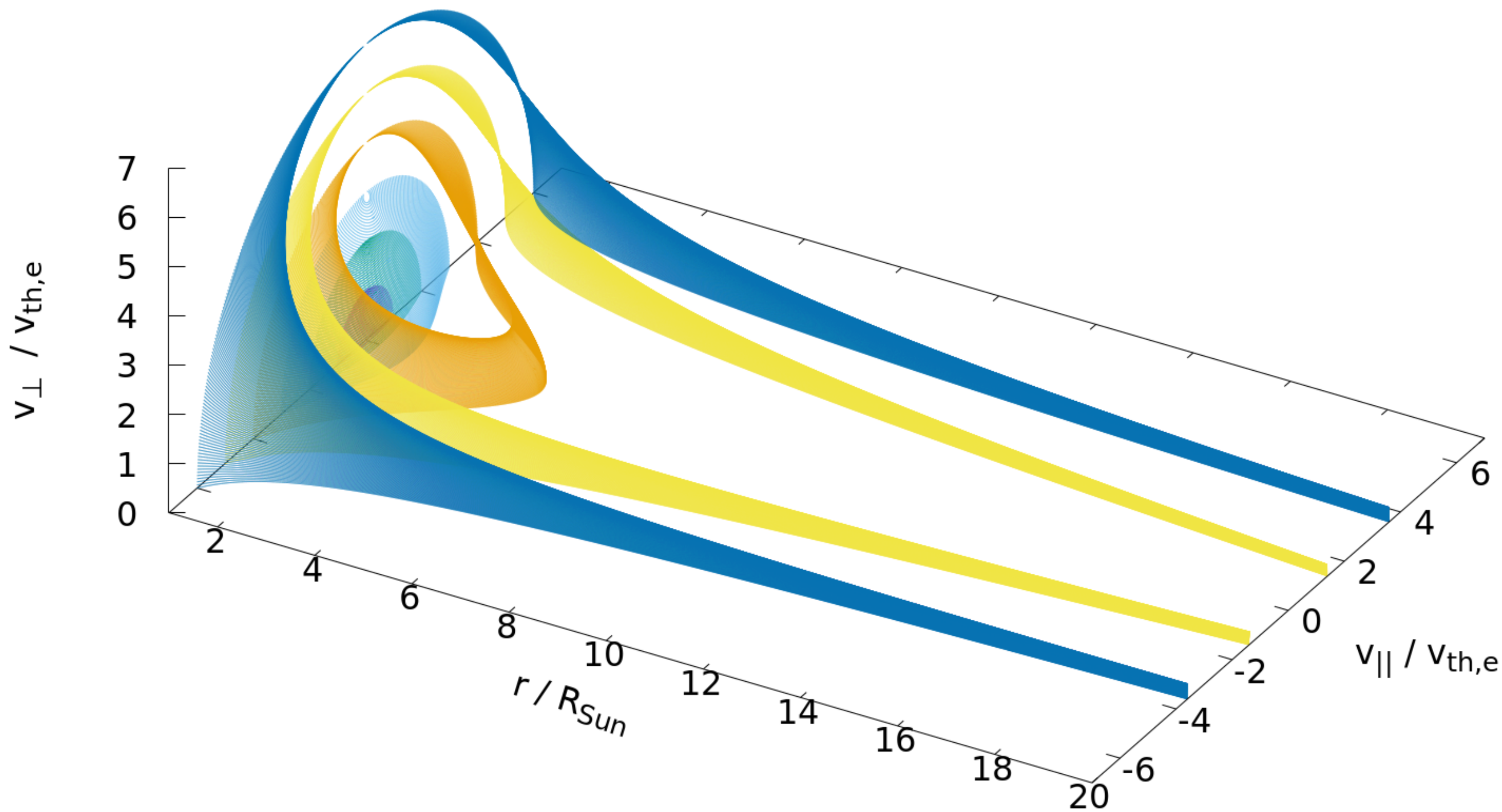
E_z/E_0
n_e
n_i
parker
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$r / R_\odot$


$\bar{g} = 9.0$, $\varepsilon = 1.0 \cdot 10^{-4}$, $T_\perp \propto r^{-2.0}$, $\kappa = 4.0$, final iteration
$v_z / v_{th,i}$
8
6
4
2
0
−2
−4
−6
−8
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$r / R_\odot$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$f_i r^2$


$\bar{g} = 9.0$, $\varepsilon = 1.0 \cdot 10^{-4}$, $T_\perp \propto r^{-2.0}$, $\kappa = 4.0$, final iteration
8
6
4
2
0
−2
−4
−6
−8
$v_z / v_{th,e}$
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$r / R_\odot$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$f_e r^2$


$\bar{g} = 9.0$, $\varepsilon = 1.0 \cdot 10^{-4}$, $T_\perp \propto r^{-2.0}$, $\kappa = 4.0$, final iteration
v_e
v_i
v_{parker}
local c_S
$v/v_{th,e}$
$r/R_\odot$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



7
6
5
4
3
2
1
0
$v_\perp$ / $v_{th,e}$
2
4
6
8
10
12
14
16
18
20
r / R_{Sun}
6
4
2
0
-2
-4
-6
$v_{||}$ / $v_{th,e}$