Arbitrary open boundary conditions for data driven MHD simulations (P49)

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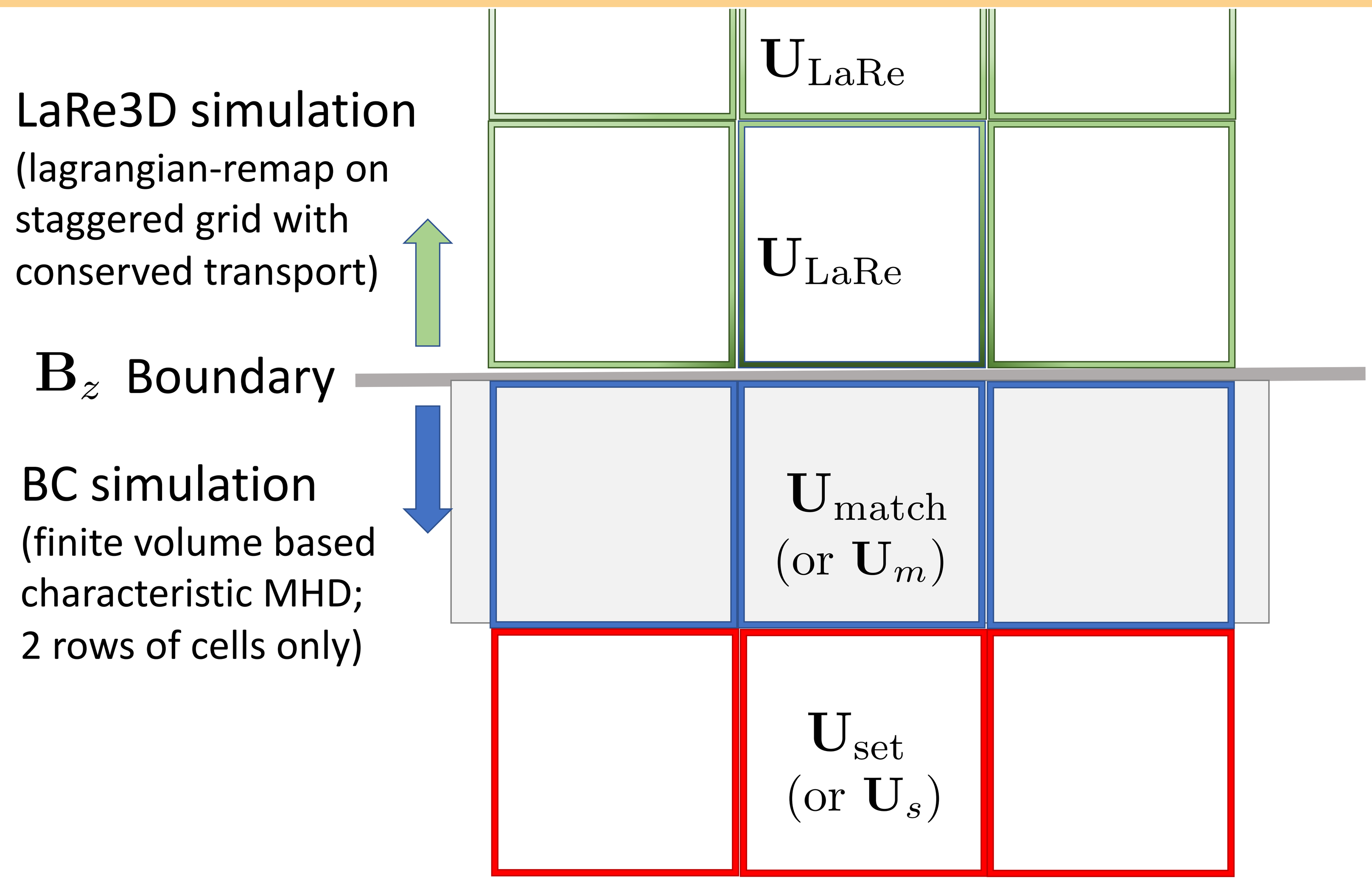
Abstract

We formulate numerical boundary conditions that incorporate a time-series of observational data. As proof of concept, we apply it to an MHD simulation of an expanding spheromak, which is a simplified model for the emergence of a solar active region. A single slice of data (the "observations") from a ground-truth LaRe3D simulation is extracted and used to data-drive a smaller simulation and validate our method. By using the characteristic form of MHD we ensure the BC does not over- or under-constrain the system. The number of inward- and outward-propagating characteristics changes with both space and time as the simulation and the observed boundary evolve. In this way, our open boundary condition (i) approximates a given time–series of observations, (ii) makes limited assumptions about the external state of the system, and (iii) is causally consistent with the internal simulation equations. The boundary itself is an actual solution to the time-integrated MHD equations. We present the validation of our method by comparing our ground truth and driven simulations, as well as highlight the strengths, weaknesses, and future extensions of our method. See also: N.D.Kee, P15 for a comparison of "nonreflecting" BCs

Background and Setup



MHD State: $\mathbf{U} = (\rho, \epsilon, v_x, v_y, v_z, B_x, B_y, B_z)$

$\mathbf{U}_{\rm LaRe}$: Regular MHD state update by the LaRe3D code

$\mathbf{U}_{\rm match}$: Sequence of extracted ("observed") target state

$\mathbf{U}_{\rm set}$: BC state. Must choose so a standard integration of the MHD equations yields the sequence of $\mathbf{U}_m$ states

All MHD: $\partial_t \mathbf{U} + \mathbb{A}_x \cdot \partial_x \mathbf{U} + \mathbb{A}_y \cdot \partial_y \mathbf{U} + \mathbb{A}_z \cdot \partial_z \mathbf{U} + \mathbf{D} = \mathbf{0}$

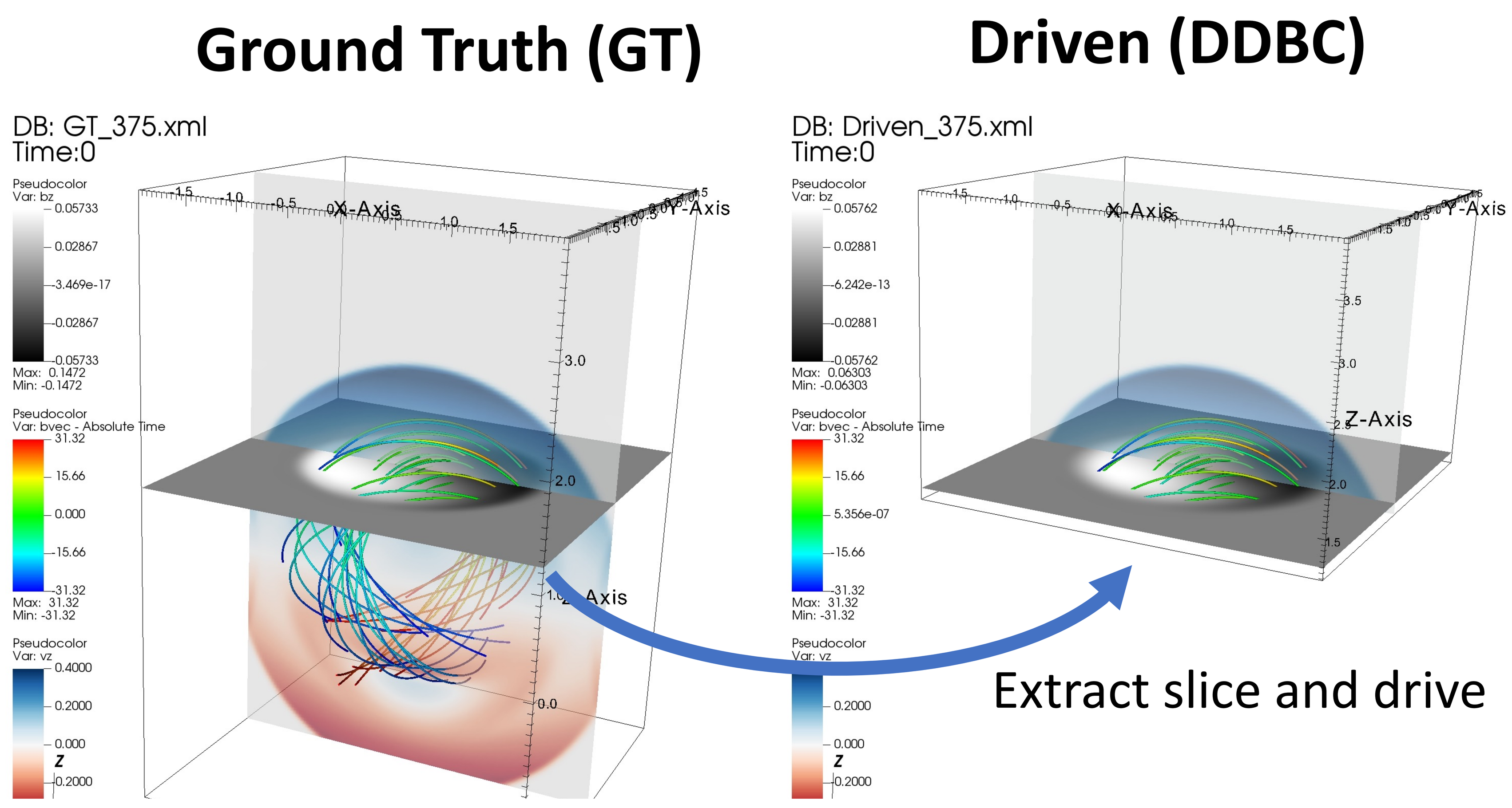
Characteristics BC: partition the coefficient matrix to break the derivative into forward and reverse propagating components:

$$\mathbb{A}_z \cdot \partial_z \mathbf{U} \longrightarrow \left(\mathbb{A}_z^+ + \mathbb{A}_z^-\right)\partial_z \mathbf{U} = \mathbf{F}_z^+ + \mathbf{F}_z^-$$

Separate terms that depend on $\mathbf{U}_s$: $\partial_t \mathbf{U}_m + \mathbf{C} + \mathbb{A}_z^+ \partial_z \mathbf{U}\big|_{l.b} = 0$ (flux at red/blue interface)

Formally, $\int \partial_z \mathbf{U} = \mathbf{U}_m - \mathbf{U}_s = -\int \left[\mathbb{A}_z^+\right]^{-1} \left(\partial_t \mathbf{U}_m + \mathbf{C}\right)$

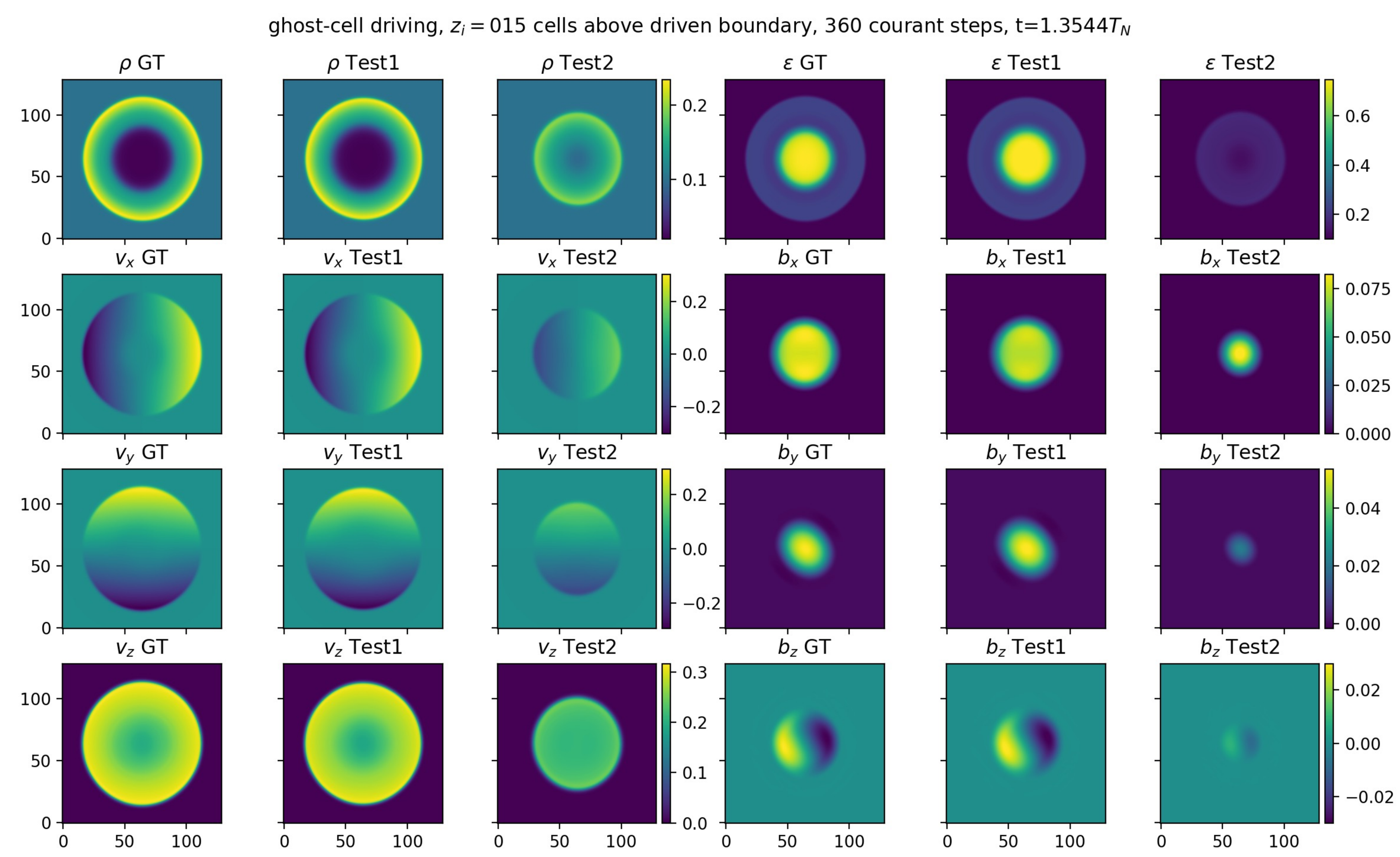
- Red variables depend on the BC state: problem is nonlinear
- Setting flux $\mathbf{F}_z^+$ directly is fraught with danger! There may be no state $\mathbf{U}_s$ consistent with that flux.



Compare 3 simulations: one GT, two tests of DDBC

Test 1: Set all variables in $\mathbf{U}_s$ straight from the GT simulation

Test 2: Set $\mathbf{U}_s$ with $(\partial_t\rho, \partial_t\epsilon, v_z)\big|_{\rm BC} = 0; \mathbf{v}_h, \mathbf{B}$ from GT



Summary

- Rigorous testing instills confidence in the driving of the MHD model with real observations
- Need reasonable estimate of MHD state vector (not just B).
- Allows coupling between different MHD codes
- Ensures the driving layer is a solution to the MHD equations
- Typical techniques for data driving like $(\partial_t\rho = \partial_t\epsilon)\big|_{\rm BC} = 0$ generate large and increasing departures from G.T.

REFERENCES

(1) Data Driven BCs for MHD Simulations: Tarr, L.A.+ 2022 ApJ (in prep) **(2) "Non-Reflecting" BCs for MHD Simulations: N.D. Kee+ 2022 ApJ (in prep)**

LARE3D: Arber+ 2001 JCP 171 ; Characteristics/BCs: Jeffery & Taniuti, 1964 "Nonlinear Wave Propagation" (book); Nakagawa 1981 ApJ 247; Thompson 1990 JCP 89; Poinsot & Lele 1992 JCP 101; Grappin+ 2000 A&A 362; Yalim+ 2017 JPCS 837; Toriumi+ 2020 ApJ 890 (simulation shootout); Jiang+ 2021 *The Innovation* (review)

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