# Towards Uncertainty Quantification for Synthetic White Light Images in the Space Weather Modeling Framework

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## Motivation
### Space Weather with Quantified Uncertainty

NextGen SWMF Project: Reliable long-term predictions of space weather events and impact, for example Coronal Mass Ejections (CMEs) require:

Accurate physics-based modelling, propagating uncertainties from model inputs and parameters and updating knowledge of parameters (DA framework with suite of observations)

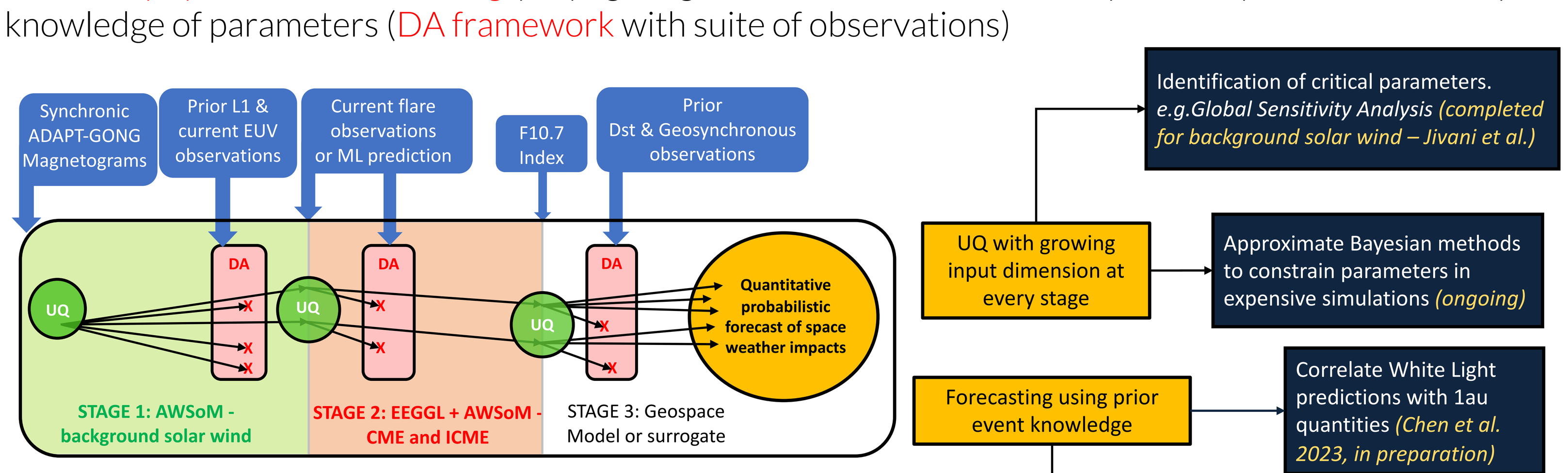



Fig 1 (a) Overall framework for NextGen SWMF
(b) Challenges and solution approaches

Stage 1: Simulations of quiet solar wind with AWSoM [van der Holst et al. (2014)] to perform Global Sensitivity Analysis (GSA) and downselect important parameters

Stage 2 (ongoing): Conduct simulations of multiple CME events with joint design of solar wind - flux rope parameters

## Workflow for CME Simulations
### EEGGL- Gibson-Low Flux Rope, AWSoM – CME Propagation

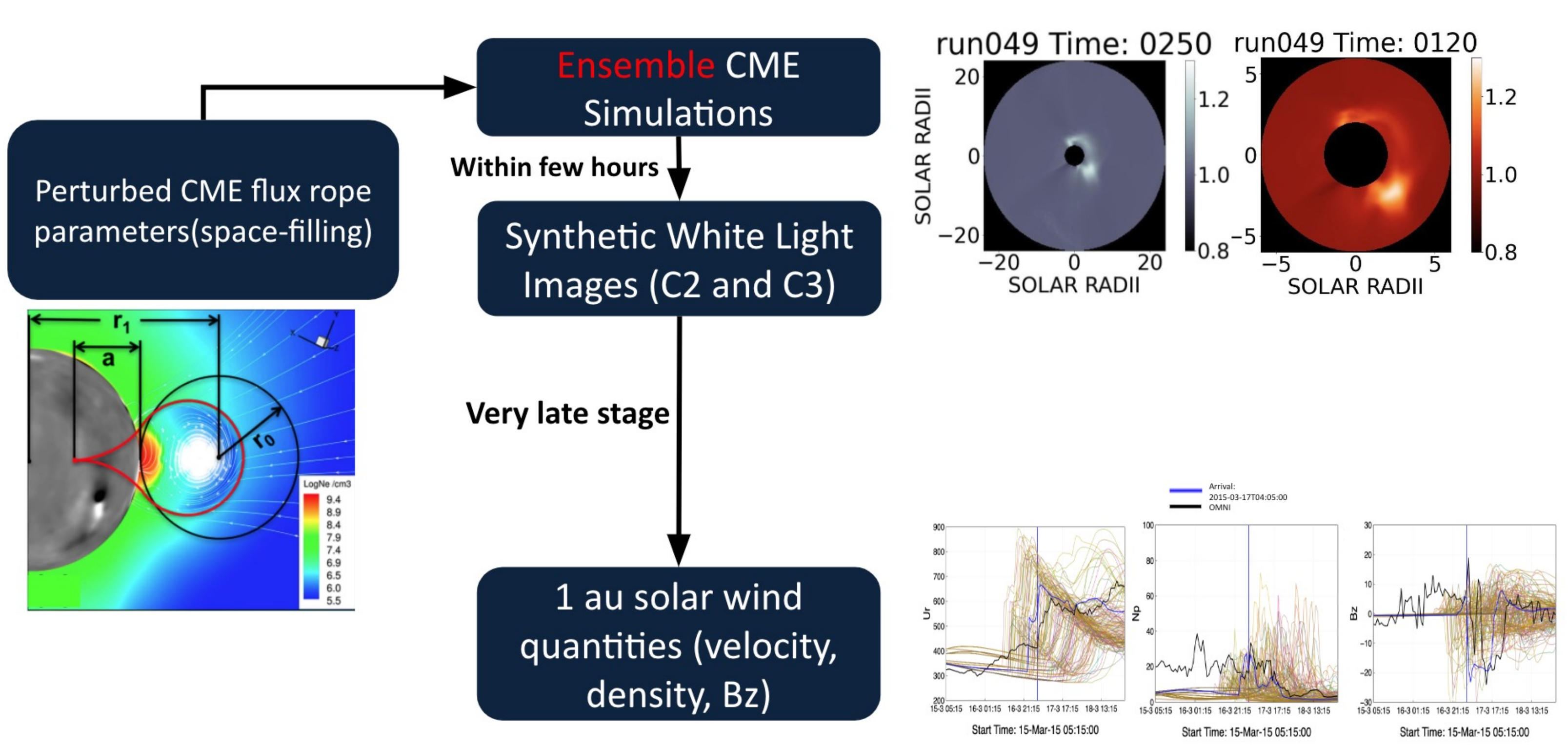



Fig 2 : Conducting Sun-to-Earth Simulations using EEGGL and AWSoM, with multiple data products (remote and in-situ) to validate simulations

## Emulating White Light Images
### First Step: POD based surrogate

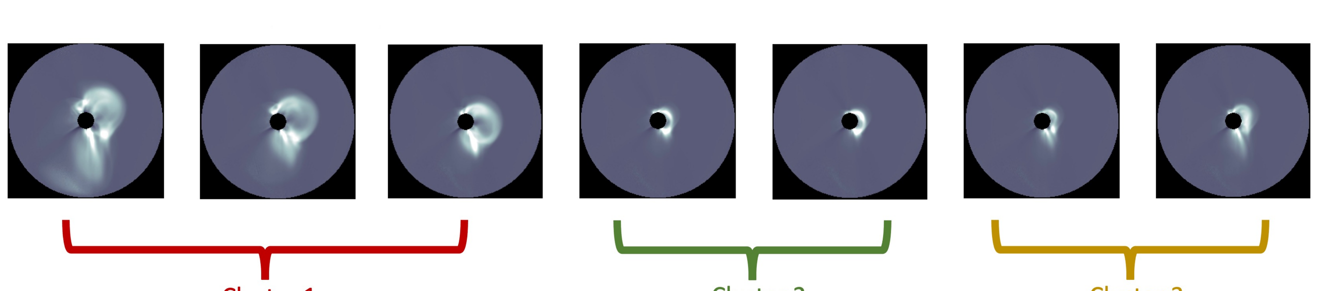
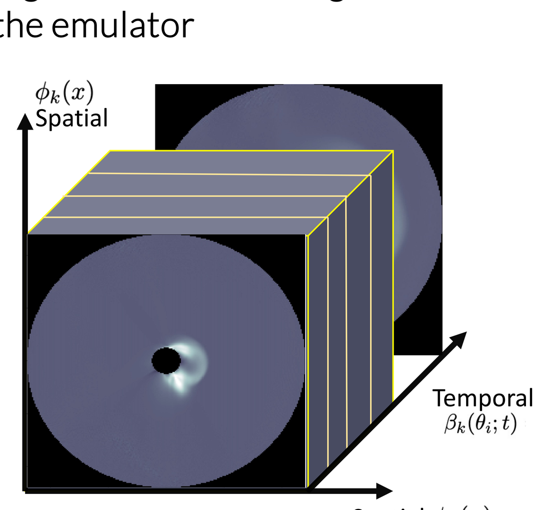



Fig 3 Clusters of image data based on the Wasserstein distance, used to create balanced training set for the emulator




General Idea of Proper Orthogonal Decomposition – describe spatio-temporal fields using linear combinations of basis functions

$$\hat{f}(x,t;\theta_i) = \sum_{k=1}^{r} \phi_k(x)\beta_k(\theta_i;t)$$

Depends on spatial coordinate $x$

Function of flux rope parameters $\theta$ and time $t$, we map to $\theta$ using Polynomial Chaos Expansions

$$= \sum_{j=1}^{n_t} \left[\sum_{k=1}^{k_t} \sqrt{\lambda_k} q_k(\mathbf{x}) b_{k,\beta^{(j)}}(\mathbf{t})\right] \Psi_{\beta^{(j)}}(\xi(\theta),\mathbf{t})$$

Eigenvalues and eigenfunctions of sample covariance

These expansions map parameters from physical space to expansion coefficients via orthogonal polynomials

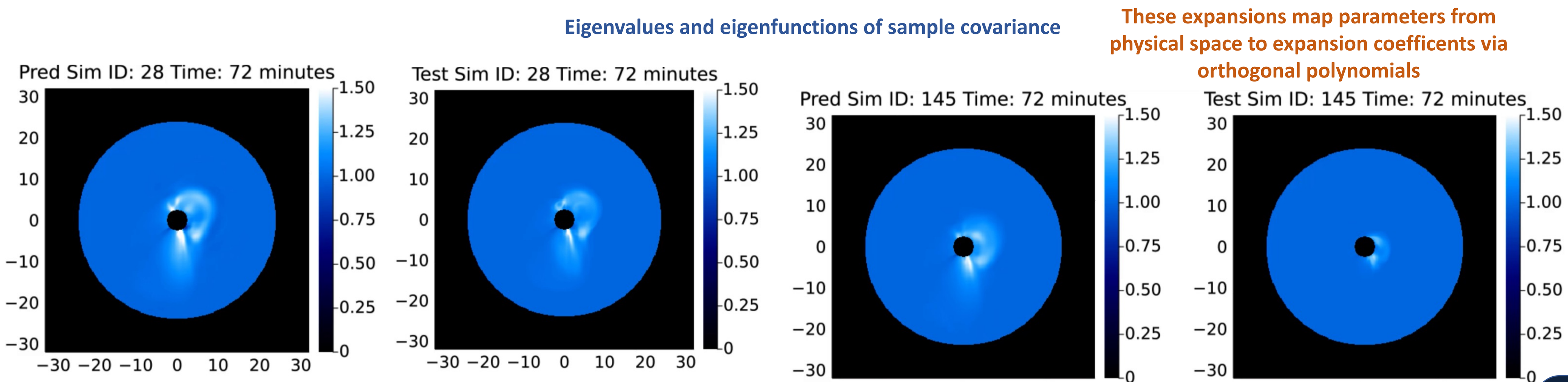



Fig 4 Sample emulator predictions vs true AWSoM simulation image for two test simulations. The emulator can capture relevant structure but carry significant bias in speed

**Advantages:**
- Computationally inexpensive (requires SVD of a moderately sized matrix and linear regression)
- Modes describe variation across parameter space (i.e. global model is learnable from available simulations)

**Drawback:**
Limited extrapolation capability on account of lack of constraints from physical model

## Method 2: Operator Inference
### Popular Approach for Projections of Dynamical Systems

Operator Inference or OpInf: [Peherstorfer and Willcox 2016, Issan and Kramer 2023]
Data-driven Reduced Order Model (ROM) – model the dynamics of a system of ODEs by inferring low dimensional operators in a non-intrusive manner

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{y}(t) = \mathbf{F}(t,\mathbf{y}(t),\mathbf{u}(t)) \xrightarrow{\text{Project to } \tilde{\mathbf{y}}} \frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{y}}(t) = \hat{\mathbf{c}} + \hat{\mathbf{A}}\hat{\mathbf{y}}(t) + \hat{\mathbf{H}}[\hat{\mathbf{y}}(t)\otimes\hat{\mathbf{y}}(t)] + \hat{\mathbf{B}}\mathbf{u}(t)$$

by any suitable method like POD (Intermediate Step)

Infer low dimensional operators via linear regression (user choice on number of polynomial terms)

For every individual simulation (C3 Coronagraph FOV):

Infer dynamics and extrapolate (linear and quadratic)

| Train Time | Predict Time |
|---|---|
| t=40 min | t=120 min — t=180 min |

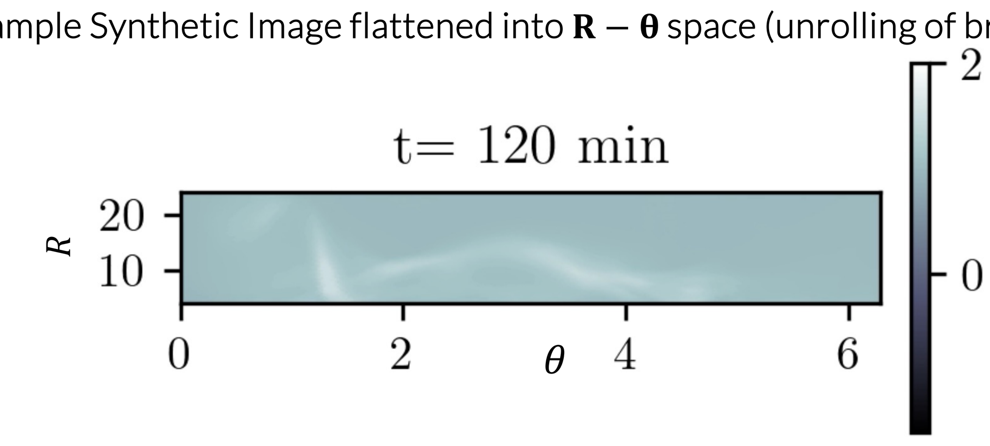
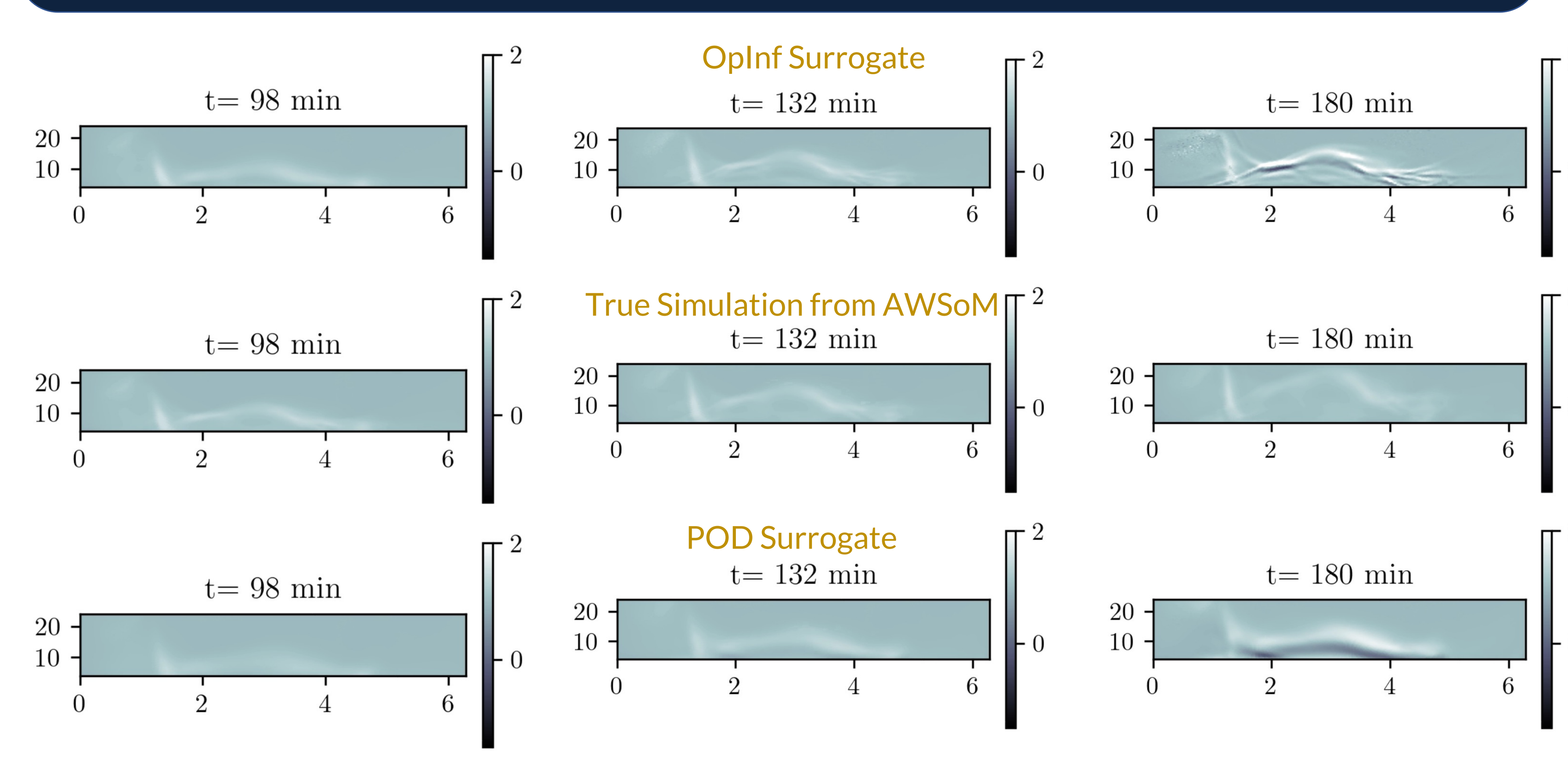



Fig 6 Sample Synthetic Image flattened into $\mathbf{R}-\boldsymbol{\theta}$ space (unrolling of bright annulus)

**Advantages:**
- Models the change in brightness as a function of time and flux rope parameters
- Can extrapolate better with limited training whereas vanilla POD errors grow quickly

**Drawback:**
- Choosing suitable model form for the ROM when approximating an unknown ODE can be difficult.
- Learning global model requires interpolation in high-dimensional operators for limited training data

## Comparisons and Next Steps
### Incorporating Better Model Constraints



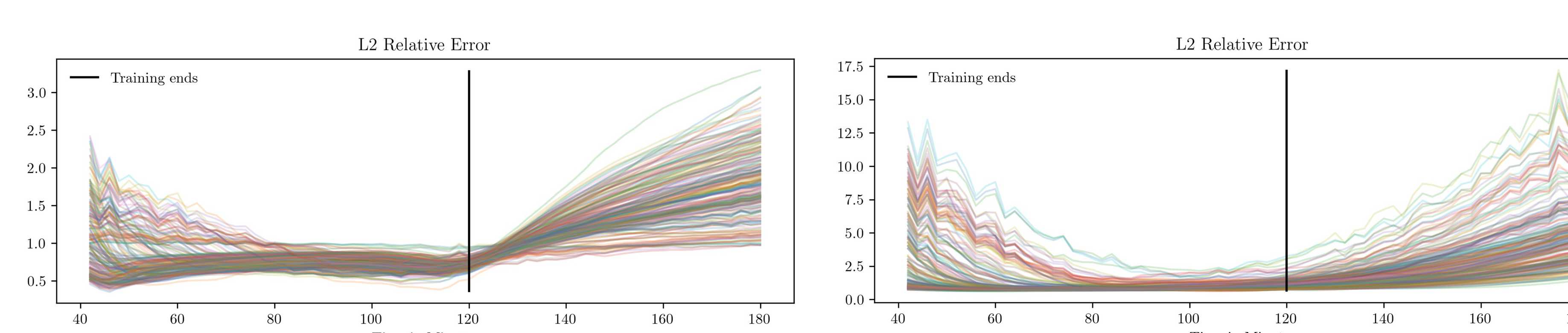




Fig 7 and 8 Operator Inference Predictions and Relative Error v/s that of POD based method. While the comparison favours OpInf, *the error is still too high to be practically useful in emulation*

**Planned Improvements:**
- Learning better reduced dimensional approximations e.g. via tensor decompositions
- Incorporating hybrid physics-ML approaches, e.g. learning operators via Neural Ordinary Differential Equations
- [UQ]: Construct prediction intervals from learnt operators to quantify uncertainties

## References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The Open Source version of the SWMF, the Michigan Sun-to-Earth Model with Quantified Uncertainties and Data Assimilation is available at: https://github.com/MSTEM-QUDA