

ELECTRON TO ION SCALE TRANSITION OF ENERGY AND ENTROPY CONVERSION IN KINETIC TURBULENCE

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MOTIVATION

- Energy conversion associated with all moments of the phase space density beyond temperature has been found to be locally significant for collisionless plasmas¹.
- This energy conversion is quantified via the so-called relative entropy/HORNET*.
- However, a detailed investigation of this energy conversion channel in magnetized turbulence has yet to be performed.
- In this study, we investigate the properties of this energy conversion in kinetic plasma turbulence using particle-in-cell (PIC) simulations of the Orszag-Tang Vortex (OTV)² in computational systems of varying domain sizes.
- The Orszag-Tang (OTV)² has been used to explore channels of energy transfer³ including the kinetic to magnetohydrodynamic (MHD) transition⁴.

* Higher-Order Non-Equilibrium Term

FIRST LAW OF KINETIC THEORY

- Recently, Cassak et al. PRL (2023)¹ generalized the energy conversion associated with all the internal moments as:

$$\frac{d}{dt} \left(\frac{s_\sigma}{n_\sigma} \right) + \frac{\nabla \cdot J_{\sigma,th}}{n_\sigma} = \frac{\dot{s}_{\sigma,coll}}{n_\sigma}$$

s_σ = kinetic entropy density
 $J_{\sigma,th}$ = thermal kinetic entropy density flux
 $\dot{s}_{\sigma,coll}$ = time rate of change of kinetic entropy density through collisions

Position space Velocity space

$$\frac{d}{dt} \left(\frac{s_{\sigma p}}{n_\sigma} \right)$$

$$\frac{d}{dt} \left(\frac{s_{\sigma v}}{n_\sigma} \right)$$

$$\frac{1}{T_\sigma} \frac{dW_\sigma}{dt}$$

$$\frac{1}{T_\sigma} \frac{dE_{\sigma,int}}{dt}$$

$$\frac{d}{dt} \left(\frac{s_{\sigma v,rel}}{n_\sigma} \right)$$

$$-\frac{1}{T_\sigma} \frac{dQ_\sigma}{dt}$$

$$\frac{(\nabla \cdot J_{\sigma,th})_{rel}}{n_\sigma}$$

Compressional work done per particle
-ve: compression
+ve: expansion

Increment in internal energy ($E_{\sigma,int}$)

$s_{\sigma v,rel}$ is the relative entropy density

Thermodynamic heating rate per particle

Thermal relative entropy density flux

Pressure dilatation

Temperature

Non-maxwellianity
HORNET

Pi-D
Divergence of heat flux

HORNET



SIMULATIONS

- Particle-in-cell (PIC) simulations of Orszag Tang Vortex (OTV) using p3d⁵ code.
- The initial magnetic (\mathbf{B}) and velocity field (\mathbf{v}) for both ions & electrons are defined as:

$$v_x = -\sin\left(\frac{2\pi y}{L_y}\right)$$

$$v_y = \sin\left(\frac{2\pi x}{L_x}\right)$$

$$v_z = 0$$

$$B_x = -\sin\left(\frac{2\pi y}{L_y}\right)$$

$$B_y = \sin\left(\frac{4\pi x}{L_x}\right)$$

$$B_z = 5$$

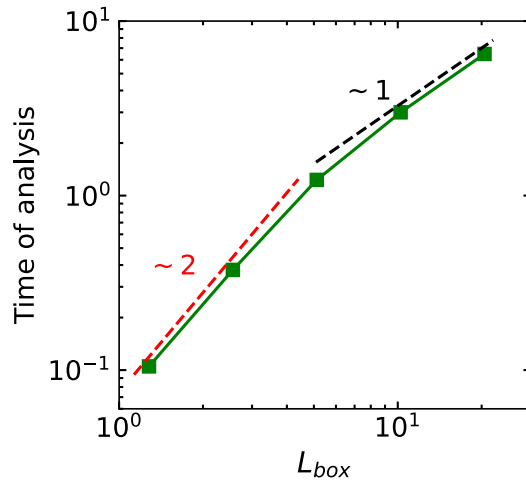
\mathbf{B} normalized to B_0
 \mathbf{v} normalized to c_{A0}

- Electrons however get an extra v_z to satisfy $\nabla \times \mathbf{B} = \mathbf{J}$ (code units).
- We use computational domains of different sizes. Here are the details:

Run	$L_{\text{box}} (d_i)$	$N_x=N_y$	δx	δt	c^2	m_e/m_i	$T_i=T_e$	N_{ppg}	λ_D	ρ_e	$1/w_{pe}$
I	1.28	128	0.01	0.00075	1600	0.04	0.5	400	0.0177	0.04	0.005
II	2.56	128	0.02	0.0015	400	0.04	0.5	400	0.0354	0.04	0.005
III	5.12	256	0.02	0.0015	400	0.04	0.5	400	0.0354	0.04	0.005
IV	10.24	512	0.02	0.0015	400	0.04	0.5	400	0.0354	0.04	0.005
V	20.48	1024	0.02	0.0015	400	0.04	0.5	400	0.0354	0.04	0.005

TIME OF ANALYSIS

- Global non-linear time estimate: $\tau_{nl} = \frac{\lambda_c}{\delta z_{rms}} = \frac{\frac{L_{box}}{2\pi}}{\sqrt{(\delta v_{rms})^2 + (\delta b_{rms})^2}}$ δz_{rms} = turbulence amplitude
 L_{box} = system size
- If systems are initiated with the same turbulence amplitude: $\tau_{nl} \propto L_{box}$
- However, time scales in smaller boxes are found to be much faster than predicted by $\tau_{nl} \propto L_{box}$.
- Time of analysis is chosen when all the systems have similar structures and the magnetic flux between X- and 0-line follow $d\psi \propto L_{box}^{-1}$.



L_{box} (d_i)	1.28	2.56	5.12	10.24	20.48
Time of analysis t_A (tw_{ci})	0.09	0.34	1.23	3.00	6.48
Nonlinear time $\tau_{nl}(tw_{ci})$	0.144	0.228	0.576	1.152	2.30
Ratio t_A/τ_{nl}	0.625	1.513	2.14	2.6	2.81

Fig.1 Scaling of the time of analysis ($t\omega_{ci}$) with the system size. Smaller systems follow $\tau_{nl} \propto L_{box}^2$ while large one follow $\tau_{nl} \propto L_{box}$.

- In a whistler wave, phase velocity is dependent of the wave number ($\sim 1/L$).
- In an Alfvén wave, phase velocity is equal to the Alfvén velocity (independent of wave number).

SYSTEM OVERVIEW

- Overview of the system at the time of analysis (see Fig. 2), where all systems have similar global structure.

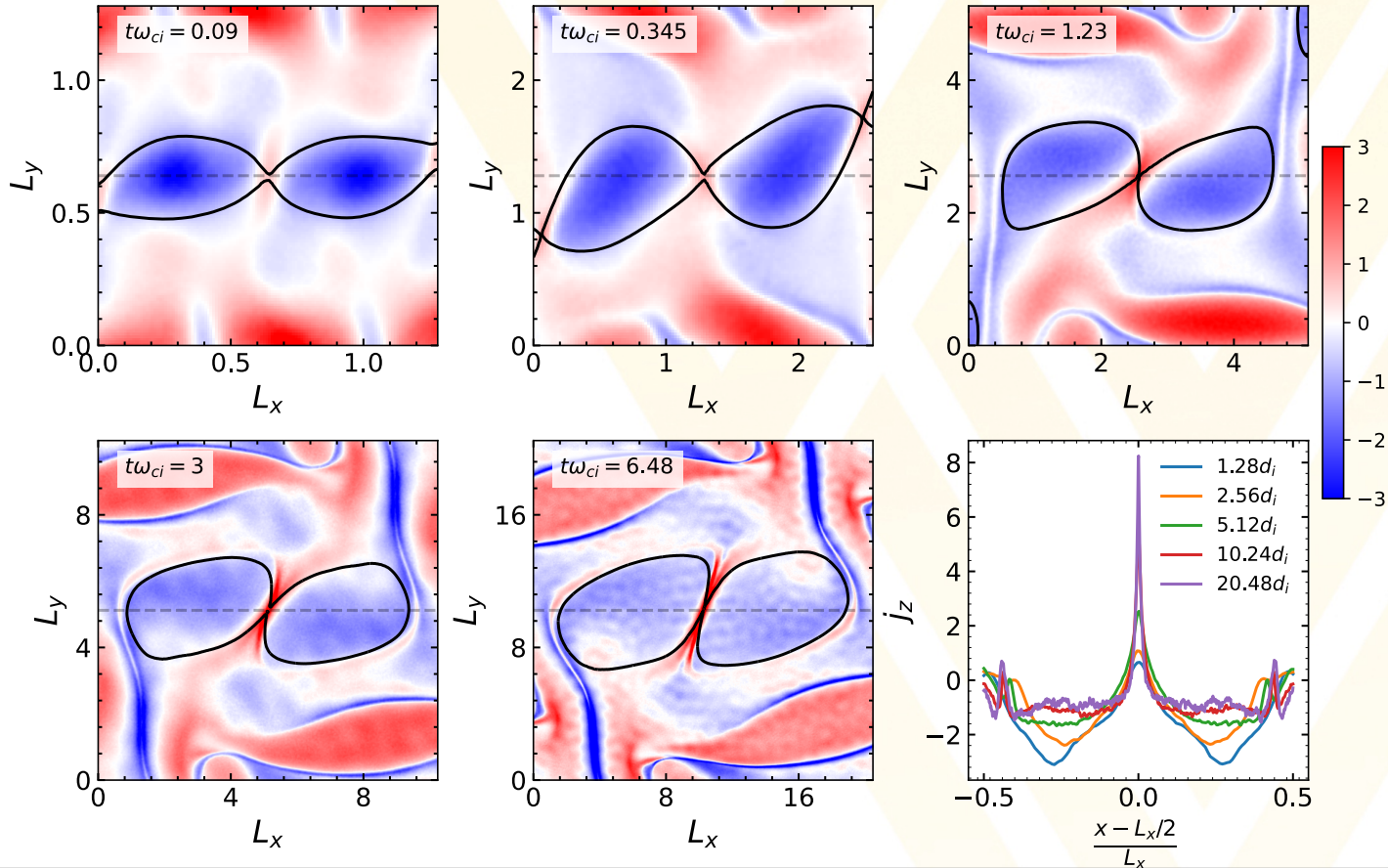


Fig.2 Out of plane current (j_z) for all the runs at the time of analysis with a contour of magnetic flux function through the central X-line. The bottom right panel shows the cut of j_z along the dotted line across each panel.

ENTROPY AND TEMPERATURE

- Entropy is correlated with temperature fluctuations (T_{\parallel} for electrons, T_{\perp} for ions)

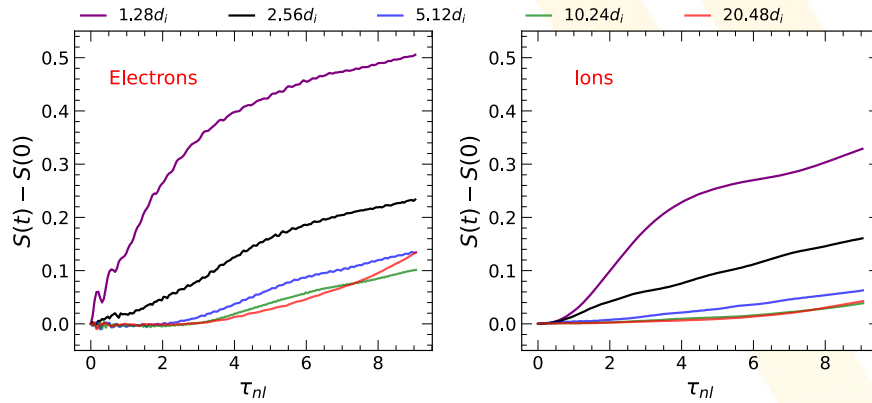


Fig. 3 Evolution of entropy for ions and electrons in all the systems as a function of τ_{nl} , the global non-linear time.

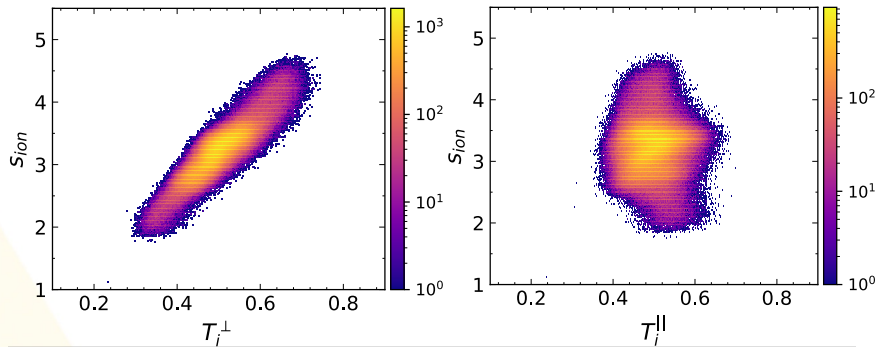


Fig. 4 Joint pdf between entropy density with parallel and perpendicular temperature for ions. Ion entropy is correlated with T_{\perp} and uncorrelated with T_{\parallel} .

Kinetic entropy density

$$s_{\sigma} = -k_B \int f_{\sigma} \ln \left(\frac{f_{\sigma} \Delta^3 r \Delta^3 v}{N} \right) d^3 v$$

f_{σ} = phase space density

$\Delta^3 r \Delta^3 v$ = phase space volume

Total kinetic entropy

$$S_{\sigma} = \int s_{\sigma} d^3 r$$

- Below are results from $L_{\text{box}} = 20.48d_i$.

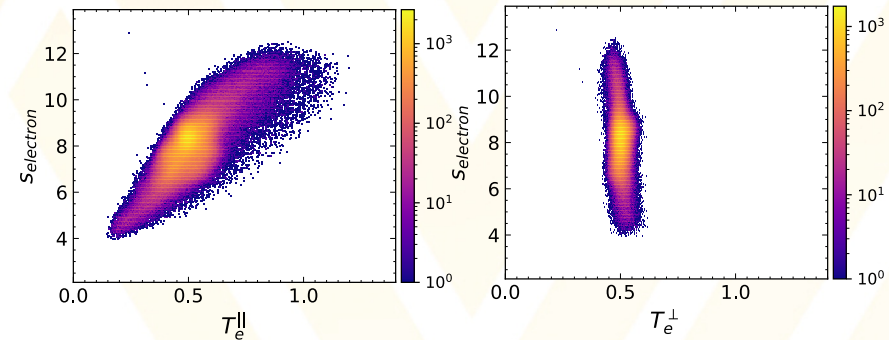


Fig. 5 Joint pdf between entropy density with parallel and perpendicular temperature for electrons. Electron entropy is correlated with T_{\parallel} and uncorrelated with T_{\perp} .

ANISOTROPY & HEATING

- Electrons are heated in \parallel direction, and ions in the \perp direction (similar to Parashar et al.⁴).
- As system size increases relative temperature (pressure) anisotropy decreases.

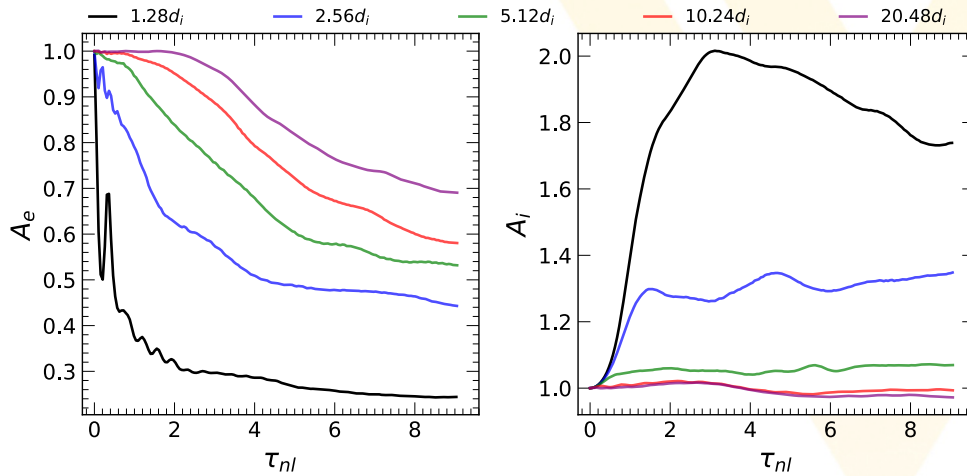


Fig. 6 Temperature anisotropy ($A=T_{\perp}/T_{\parallel}$) for all the simulations.

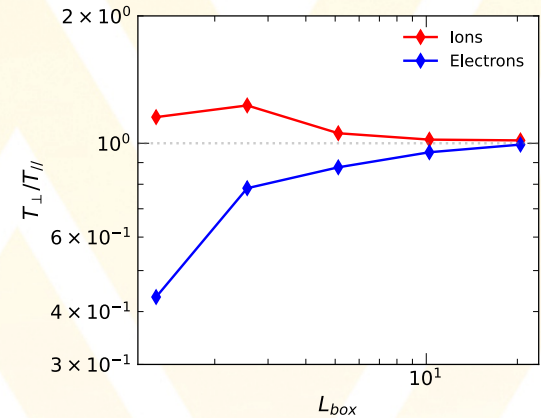


Fig. 7 Scaling of temperature anisotropy at the time of analysis.

- Ion and electron heating ($Q_{\sigma} = \langle \partial_t E_{\sigma}^{th} \rangle$) scales as L_{box}^{-2} .
- Physical scales of ion-electron dynamics unchanged when L_{box} increases.
- Q_i/Q_e fairly independent of the system size except for the smallest box ($1.28d_i$), where ions are least coupled.

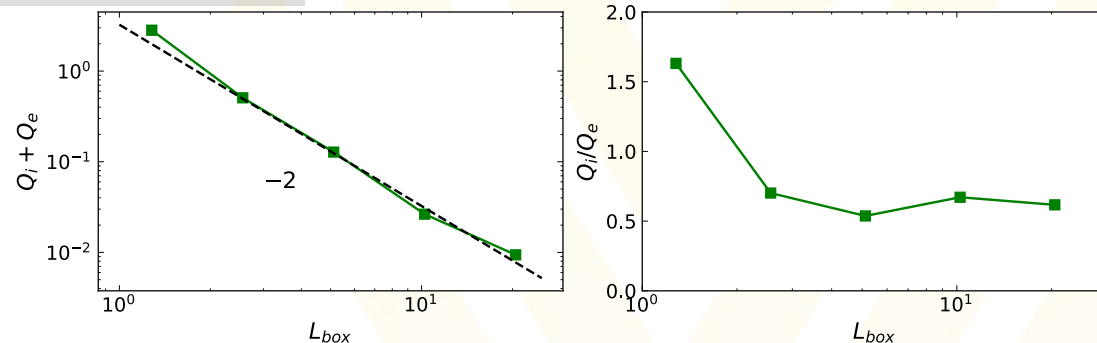


Fig. 8 Scaling of ion plus electron heating (left) and their ratio (right) at the time of analysis.

RELATIVE ENTROPY

- Relative entropy ($S_{\sigma v,rel}$) measures deviation from Maxwellianity as it contains physics of 2nd and higher order moments.

$$\frac{S_{\sigma v,rel}}{n_{\sigma}} = -k_B \int \frac{f_{\sigma}}{n_{\sigma}} \ln \left(\frac{f_{\sigma}}{f_{\sigma M}} \right) d^3v$$

f_{σ} = phase space density

$f_{\sigma M}$ = “Maxwellianized” phase space density

- Smaller systems become highly non-Maxwellian quickly and have more average non-Maxwellianity than bigger systems.

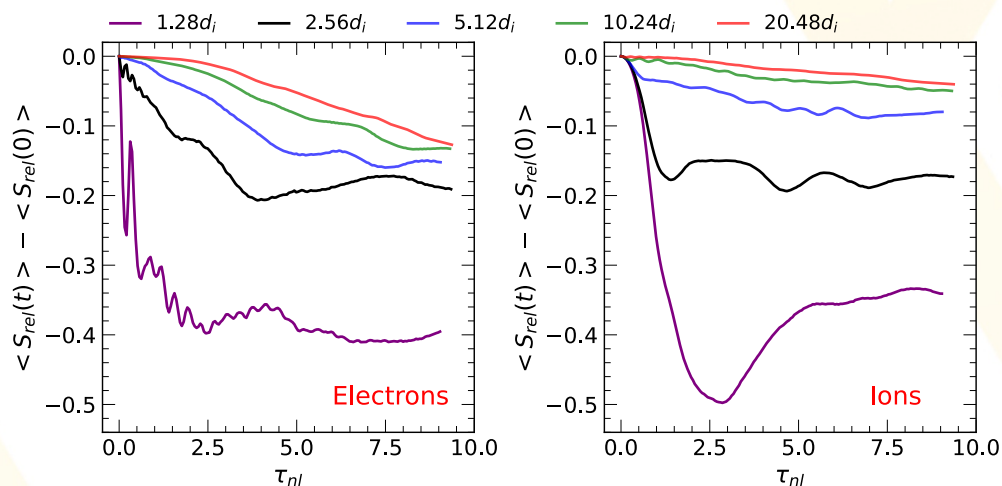


Fig. 9 Time evolution of the relative entropy for electrons (left) and ions (right) across all the systems sizes.

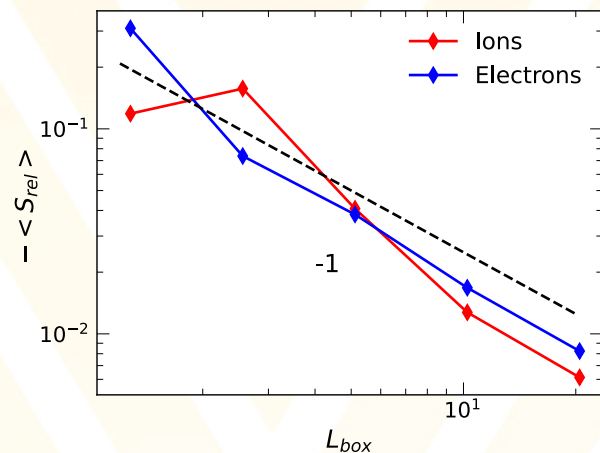


Fig. 10 Scaling relative entropy as a function of system size at the time of analysis

PRESSURE-STRAIN (PS) INTERACTION

- Pressure-strain interaction is the route of energy exchange⁶ between flow E^f and thermal energy E^{th} . For any species σ , the time evolution of different forms of energy is given by:

➤ Flow energy

$$\partial_t \langle E_\sigma^f \rangle = \langle (\mathbf{P}_\sigma \cdot \nabla) \cdot \mathbf{u}_\sigma \rangle + \langle n_\sigma q_\sigma \mathbf{u}_\sigma \cdot \mathbf{E} \rangle$$

➤ Thermal energy

$$\partial_t \langle E_\sigma^{th} \rangle = -\langle (\mathbf{P}_\sigma \cdot \nabla) \cdot \mathbf{u}_\sigma \rangle$$

➤ Electromagnetic energy

$$\partial_t \langle E^m \rangle = -\langle \mathbf{j} \cdot \mathbf{E} \rangle$$

Pressure-strain Interaction

$$-\langle (\mathbf{P}_\sigma \cdot \nabla) \cdot \mathbf{u}_\sigma \rangle = -\langle p_\sigma (\nabla \cdot \mathbf{u}_\sigma) \rangle - \langle \Pi_{\sigma,ij} D_{\sigma,ij} \rangle$$

$$p = \frac{1}{3} P_{ii} \quad (\text{scalar pressure})$$

$$\Pi_{\sigma,ij} = P_{ij} - p\delta_{ij} \quad (\text{deviatoric pressure})$$

$$\theta = \nabla \cdot \mathbf{u}_\sigma \quad (\text{velocity divergence})$$

$$D_{\sigma,ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i) - \frac{1}{3} \theta \delta_{ij} \quad (\text{traceless strain-rate tensor})$$

$$= -\underbrace{\langle p_\sigma \theta_\sigma \rangle}_{\text{Pressure dilatation}} - \underbrace{\langle \Pi_{\sigma,ij} D_{\sigma,ij} \rangle}_{\text{Pi-D}}$$

Pressure dilatation

Pi-D

PRESSURE-STRAIN (PS) INTERACTION

- Global PS interaction decreases with L_{box} (physical scales for electron-ion dynamics unchanged).

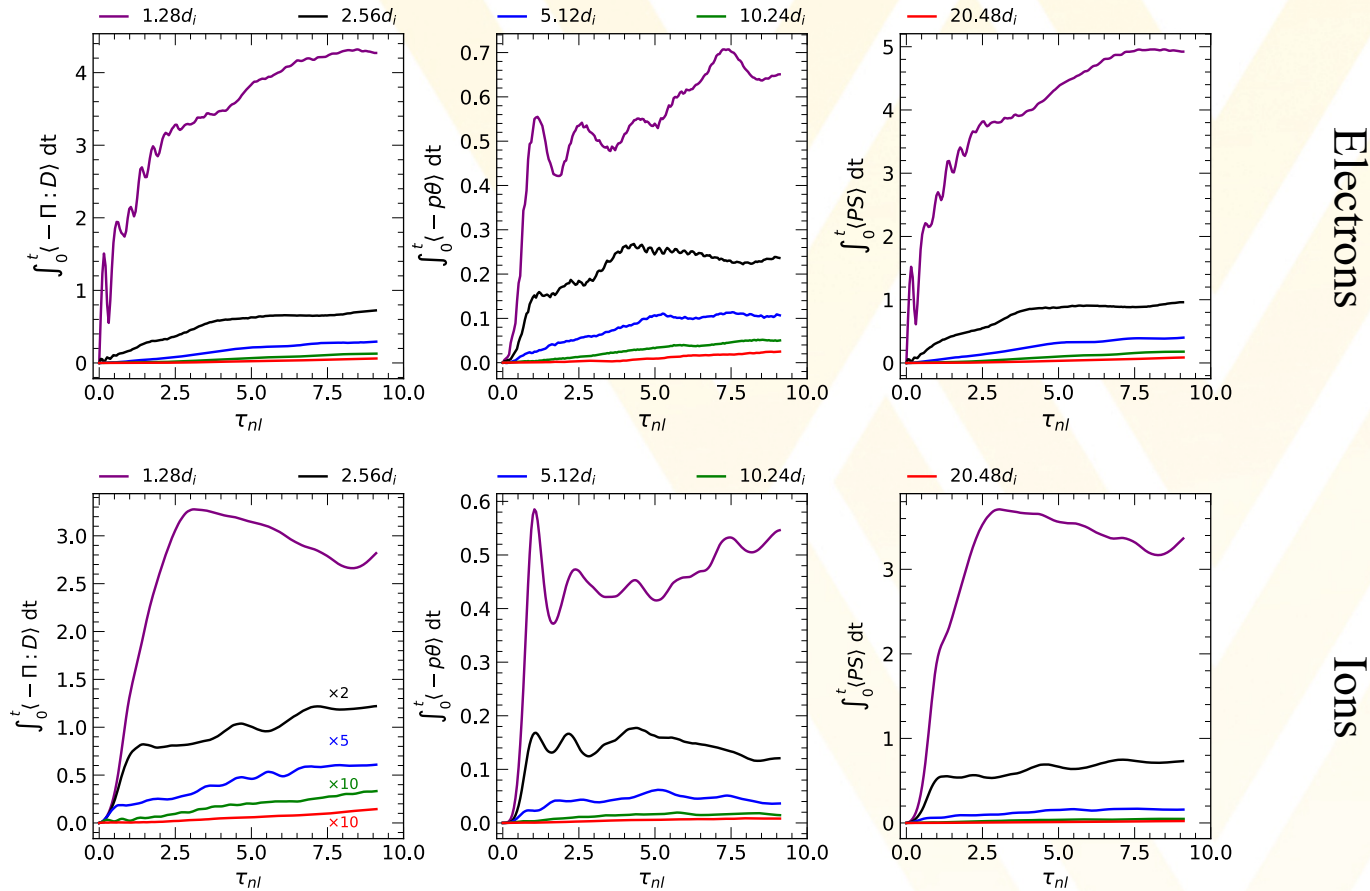


Fig. 11 Time evolution of the time integral of the box-averaged Pi-D (left panel), pressure dilatation (middle panel) and pressure-strain interaction (right panel) for electrons (top) and ions (bottom).

SCALING OF PS INTERACTION

- Pressure-strain, along with its decomposition (Pi-D and $p\theta$) scales as L_{box}^{-2} .

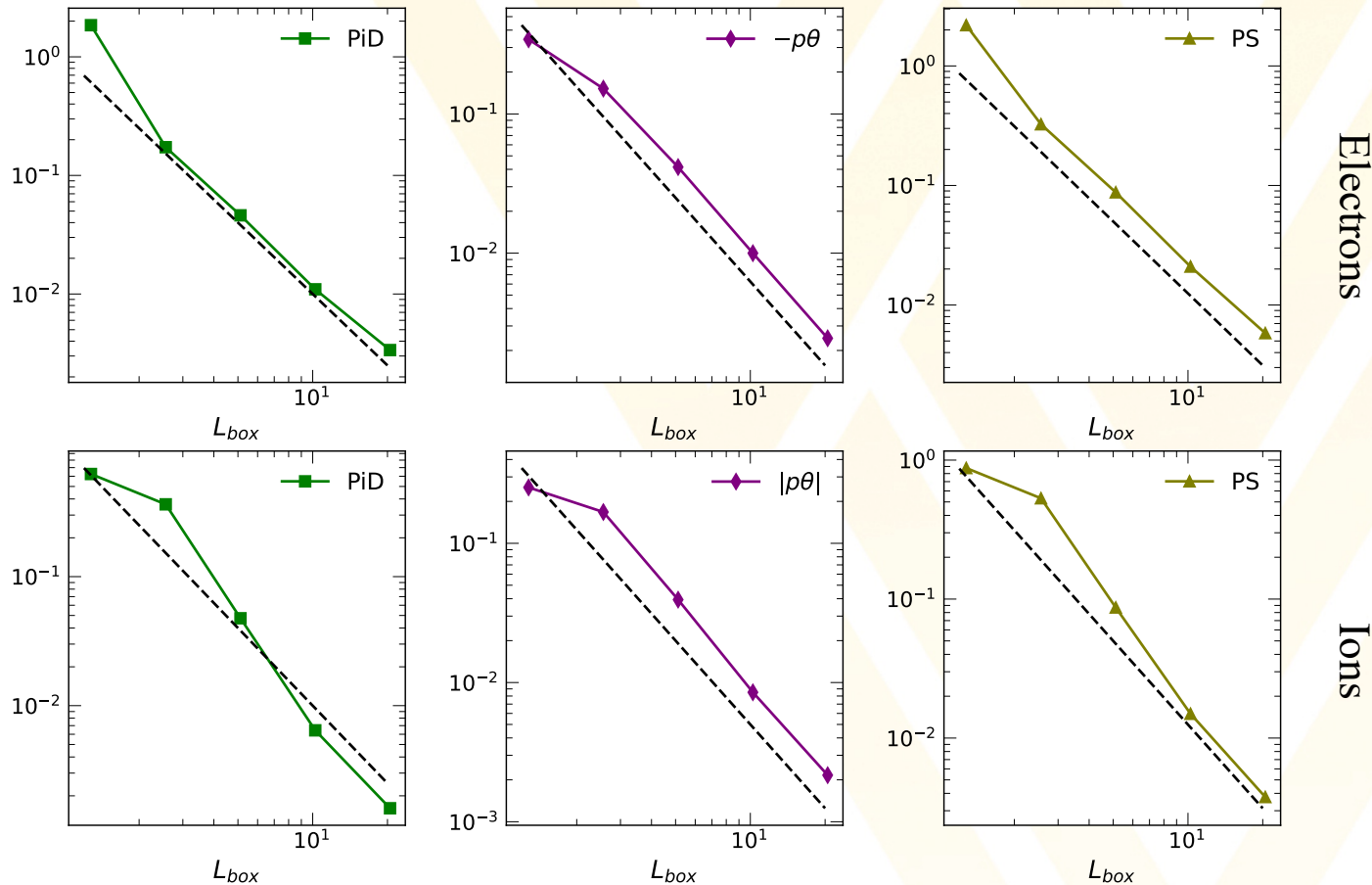


Fig. 12 Scaling of the time integral of Pi-D (left panel), pressure dilatation (middle panel) and pressure-strain interaction (right panel) for electrons (top) and ions (bottom) with the system size at the time of analysis.

ANALYSIS OF THE TERMS IN THE FIRST LAW

- How does the position space entropy relate with the velocity space entropy?
- Is the thermal kinetic entropy flux correlated with the compressible work?

Electrons ($L_{\text{box}} = 20.48d_i$)

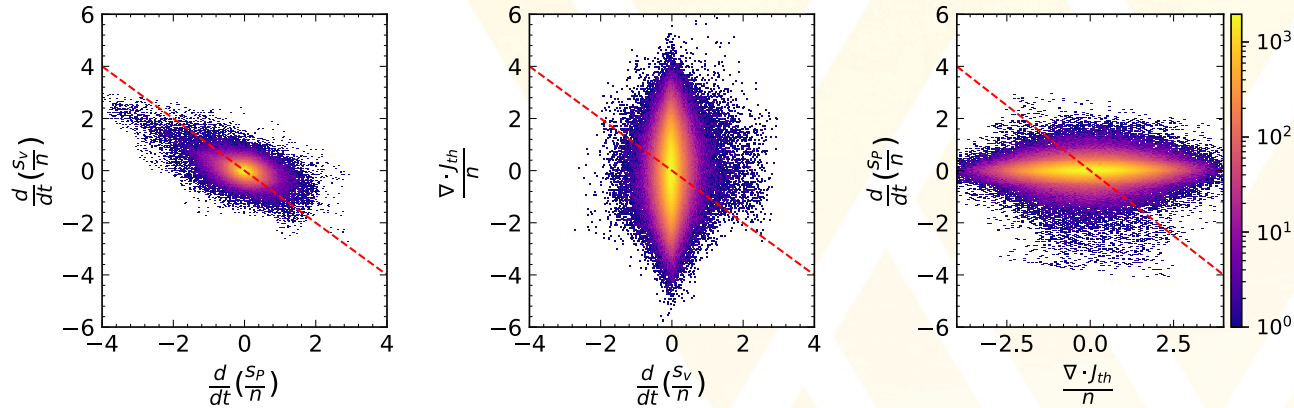


Fig. 13 Joint pdf between position space entropy and velocity space entropy (left), thermal entropy flux and velocity space entropy (middle), and position space entropy and thermal entropy flux (right) for electrons.

- Generalized work is negatively correlated with generalized internal energy (Compression leads to heating and expansion leads to cooling).
- Generalized heat uncorrelated with generalized internal energy and generalized work.

ANALYSIS OF THE TERMS IN THE FIRST LAW

- How is the relative entropy related with the generalized work?

Electrons ($L_{\text{box}} = 20.48d_i$)

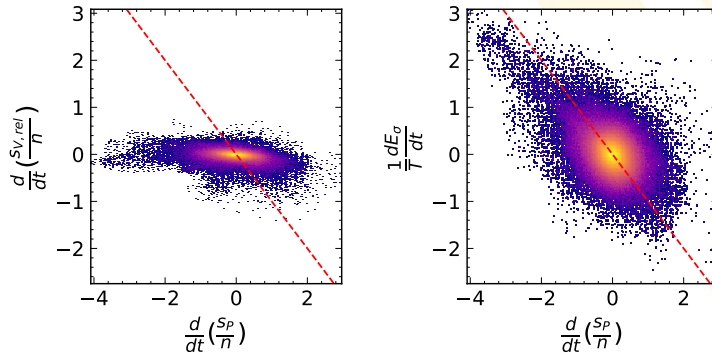


Fig. 14 Joint pdf between position space entropy and two different parts of the velocity space entropy.

- A scale filtering technique reveals that the anticorrelation between generalized work and internal energy increases with scales.
- At larger scales, $\frac{\nabla \cdot J_{th}}{n}$ becomes positively correlated with $\frac{d}{dt} \left(\frac{s_v}{n_\sigma} \right)$ and negatively correlated with $\frac{d}{dt} \left(\frac{s_p}{n_\sigma} \right)$.

- Generalized work is uncorrelated with the relative entropy.
- Generalized work anti correlated with the internal energy.

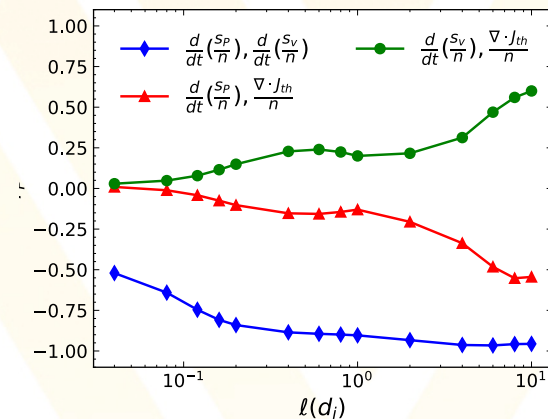


Fig. 15 Correlation coefficient observed in Fig. 11 as a function of lag scales analyzed using scale filtering.

ANALYSIS OF THE TERMS IN THE FIRST LAW

Ions ($L_{\text{box}} = 20.48d_i$)

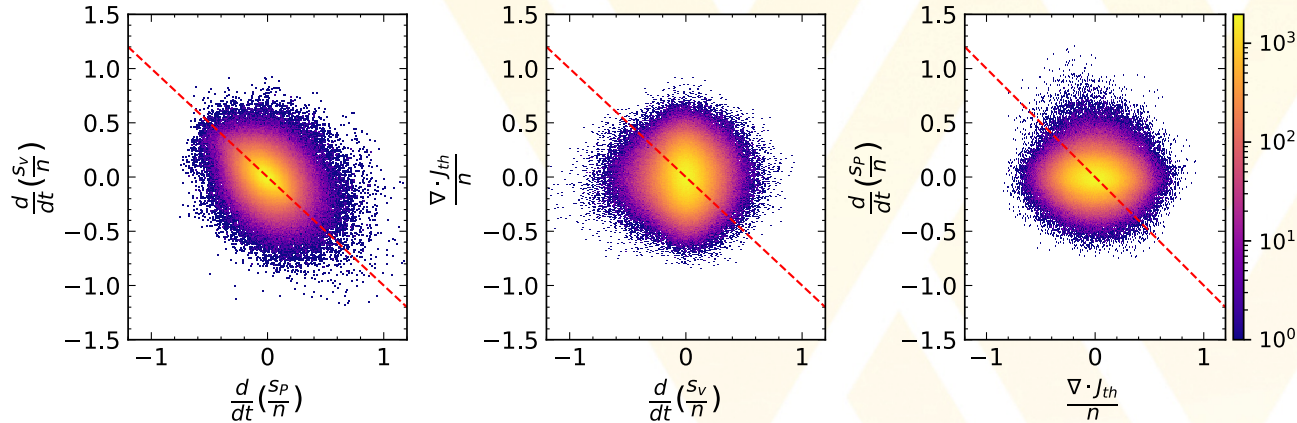


Fig. 16 Joint pdf between position space entropy and velocity space entropy (left), thermal entropy flux and velocity space entropy (middle), and position space entropy and thermal entropy flux (right) for ions.

- For ions, the correlation behavior between different terms of the first law is not as pronounced as electrons.
- The generalized work, internal energy and heat for ions seems to fluctuate less compared to the electrons.

ANALYSIS OF THE TERMS IN THE FIRST LAW

Ions ($L_{\text{box}} = 20.48d_i$)

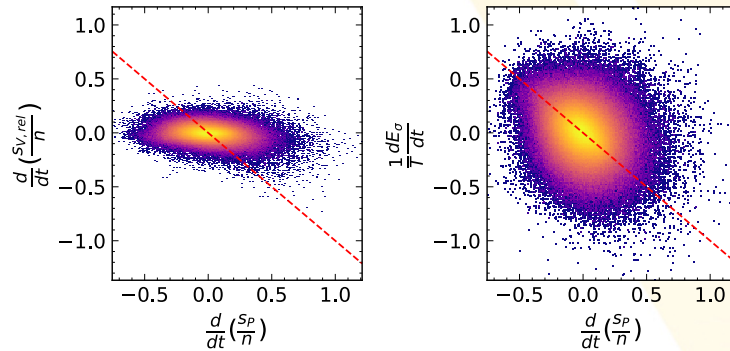


Fig. 17 Joint pdf between position space entropy and two different parts of the velocity space entropy.

- Correlation coefficient as a function of scales reveal that ions and electrons display similar behavior with the first law of kinetic theory.

- The anticorrelation between generalized work and generalized internal energy is solely due to the internal energy and not relative entropy.

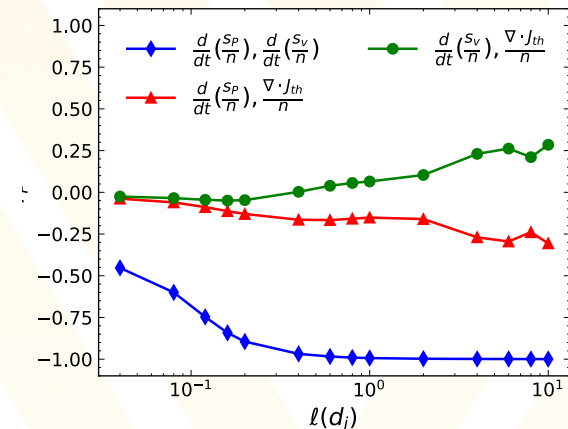


Fig. 18 Correlation coefficient observed in Fig. 14 as a function of lag scales using scale filtering.

CONCLUSION

- Entropy for ions is correlated with perpendicular temperature, entropy for electrons is influenced by the parallel temperature.
- Larger systems display smaller global anisotropy.
- Electrons and ions in smaller systems become non-Maxwellian immediately.
- Ensemble averaged pressure-strain interaction scales as L_{box}^{-2} while the average relative entropy scales as L_{box}^{-1} similar to the mean square current density (not shown) in the system.
- For most of the quantities studied, the electron-to-ion scale transition occurs for system size $\geq 2.56d_i$.
- Position space entropy (compressible work) is found to be anticorrelated with the velocity space entropy and mostly influenced by the internal energy.

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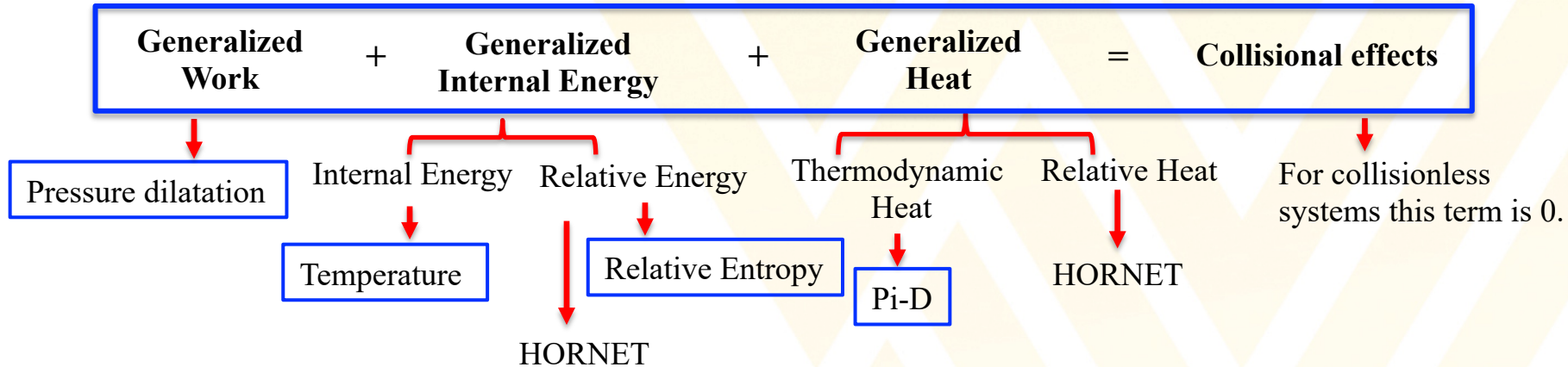
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Electron to Ion Scale Transition of Energy and Entropy Conversion In Kinetic Turbulence



- **Motivation:** Energy conversion associated with all moments of the phase space density is given by the first law of kinetic theory (Cassak+ PRL 2023).



- **Simulations:** Kinetic particle-in-cell simulations of Orszag Tang Vortex (OTV) systems with system size varying from $1.28d_i$ to $20.48d_i$.
- **Findings:**
 - Quantities such as pressure dilatation, Pi-D, temperature (heating) scales as L_{box}^{-2} .
 - Quantities such as mean square current, relative entropy, HORNET scales down as L_{box}^{-1} .
 - The ion to electron scale transition occurs starting at a system size of $L=2.56d_i$.
 - Generalized work is anticorrelated to internal energy and almost uncorrelated with relative entropy.

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