A Quantitative Comparison of Multipoint Magnetic Field Reconstruction Methods

Theodore Broeren, Kristopher Klein

University of Arizona: Applied Mathematics, Lunar & Planetary Laboratory

I. Project Overview

The upcoming NASA mission HelioSwarm (Klein et al. (2023)) will use nine spacecraft to make simultaneous measurements of space plasmas with separations spanning characteristic plasma scales. The space plasma community will use the resulting in-situ magnetometer measurements to reconstruct the magnetic field in regions near the spacecraft. We provide a comparison of common and novel reconstruction techniques that could be applied to this forthcoming data by generating synthetic spacecraft observations from a numerical simulation of turbulence. This work is intended to complement theoretical analyses of each of these reconstruction methods through application to more realistic systems, as theoretical analysis traditionally only consider smooth, non-turbulent, magnetic field structures.

This comparison will quantify the topological (macroscopic) accuracy of the reconstructed fields using each method. As it may be desirable for a reconstruction method to be able to build magnetic fields with the same distribution of small-scale fluctuations as the underlying magnetic field, we also quantify how well each of these techniques reproduces the scale-dependent statistical (microscopic) properties of the turbulent field.

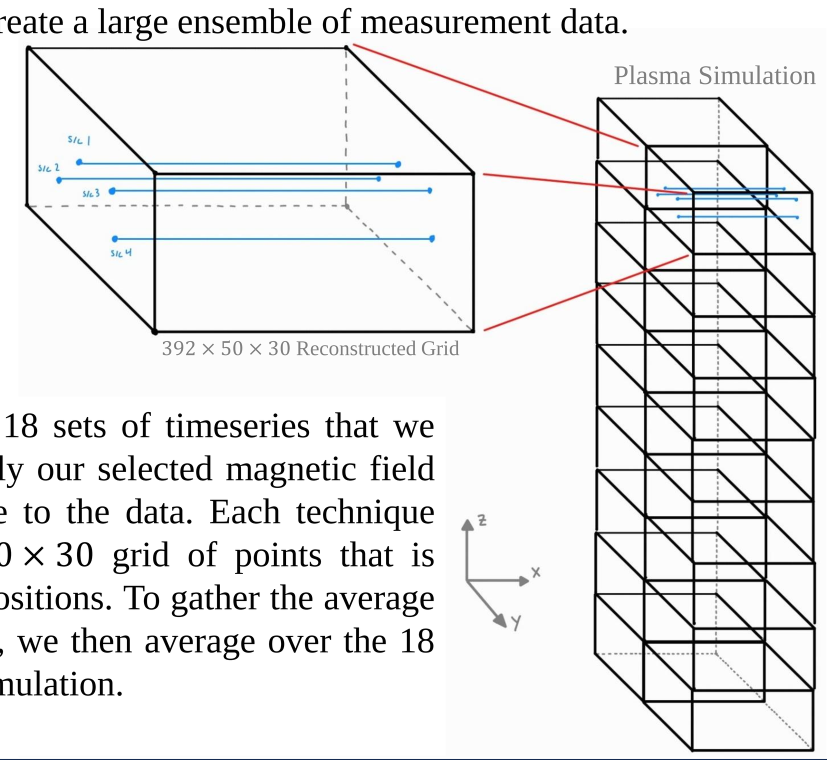
With these analyses we hope to determine which reconstruction method can most accurately, in both the macroscopic and microscopic sense, reproduce a turbulent magnetic field. We also hope to continue this work so that we can conclude if this selection is dependent on the spacecraft configuration geometry, the number of spacecraft in the configuration, or some physical characteristic of the underlying plasma turbulence.

II. Simulation Setup

As we wish to evaluate the accuracy of the magnetic field reconstruction methods in an environment that is similar to that which will be found when sampling turbulent space plasmas, we use a numerical simulation of turbulence. This simulation of turbulence is designed to have properties that mirror those found in the pristine, near-Earth solar wind. This is achieved by simulating a proton-electron plasma within the five-moment, two-fluid solver of the Gkeyll framework (Hakim et al. (2006)). See Broeren et al. (2021) for more plasma simulation details.

We let our spacecraft travel with a constant velocity of $v_{sc} = 320\hat{x}$ to approximate the solar wind traveling past a relatively-stationary spacecraft configuration. We set the sampling rate of our synthetic spacecraft measurements to 4 Hz so that the spacecraft travel a distance (80 km) greater than the grid scale of the plasma simulation (70.1 km) between consecutive measurements.

For this preliminary work, we have selected one well-shaped four-spacecraft configuration to analyze. This configuration has an elongation of 0.05, planarity of 0.01, and has an overall characteristic size of 2000km (see Paschmann (1998) for definitions). We run this configuration through the box of synthetic turbulence, traveling in the $\hat{x}$ direction, and generate magnetic field timeseries of length 392 for each of the four spacecraft. We also track the magnetic field on a grid of 50×30 points in the yz-plane surrounding the spacecraft configuration at each time step to use as our ground truth for comparison with our future reconstructions. We then repeat this pass though the plasma cube 18 times (each iteration is an independent portion of the simulation domain) to create a large ensemble of measurement data.



For each of the 18 sets of timeseries that we have generated, we apply our selected magnetic field reconstruction technique to the data. Each technique reconstructs a $392 \times 50 \times 30$ grid of points that is local to the spacecraft positions. To gather the average error from each method, we then average over the 18 iterations through the simulation.

III. Reconstruction Methods

We have implemented three magnetic field reconstruction techniques that we will compare using numerical simulations. For each of these techniques, we have assumed that Taylor's Hypothesis holds. Therefore, given N spacecraft and T time samples from each, we have NT measurements that are from the same plasma, taken from different spatial locations. The reconstruction methods that we are comparing are as follows:

(A) Curlometer

We reformulate the classic Curlometer method of current estimation to estimate the magnetic field at a desired point r (see Broeren et al (2021)). This reformulation is an estimation of the magnetic field from individual measurements at four spacecraft via a Taylor Approximation:

$$\widehat{B}_m^{(i)} \approx B_m + \sum_{k\in\{x,y,z\}} \partial B_{km} r_k^{(i)} \quad \forall i \in \{1,2,3,4\}, m \in \{x,y,z\}$$

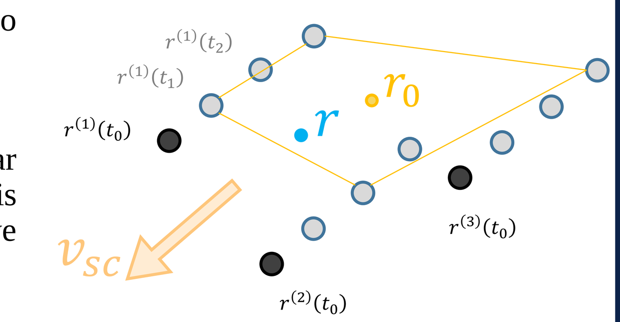
The above equation can be reformatted into a 12×12 linear system and solved exactly.

By assuming Taylors Hypothesis, we have a set of NT measurements to draw our tetrahedra from. This lets us to construct $\binom{NT}{4}$ tetrahedra, rather than the $\binom{N}{4}$ than is drawn from a static view of an N-spacecraft configuration. However, this number of possible combinations is combinatorically large, and grows super-exponentially fast! We therefore must constrain the number of tetrahedra that are considered for each reconstructed point.

Following the work developed in Broeren et al. (2021), we use a shape and location threshold to restrict ourselves to well-shaped spacecraft tetrahedra

$$\sqrt{E^2 + P^2} < 0.6$$

who have a barycenter, r_0, located near $|r_0 - r|_2 < L$ the reconstructed point r. This is the method $M_{1,3}$ described in the above reference.



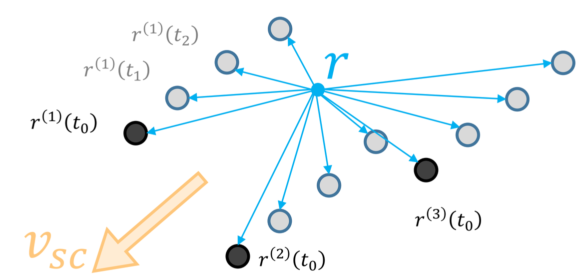
(B) RBF

The radial basis function (aka RBF; see Press et al (2007)) method uses all the magnetic field measurements to reconstruct the magnetic field at point r. Each spacecraft i's measurement at time t is weighted as a function of distance away from the reconstructed point

$$B(r) = \sum_{i=1}^{N} w_i \varphi\left(\left|r - r^{(i)}\right|_2\right).$$

Common radial basis functions are:

- Gaussian: $\varphi(r) = e^{-(\epsilon r)^2}$
- Multiquadric: $\varphi(r) = \sqrt{1 + (\epsilon r)^2}$
- Inverse: $\varphi(r) = \frac{1}{1+(\epsilon r)^2}$

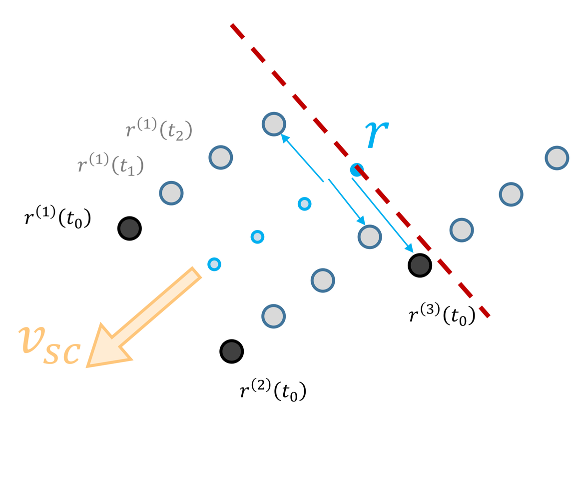


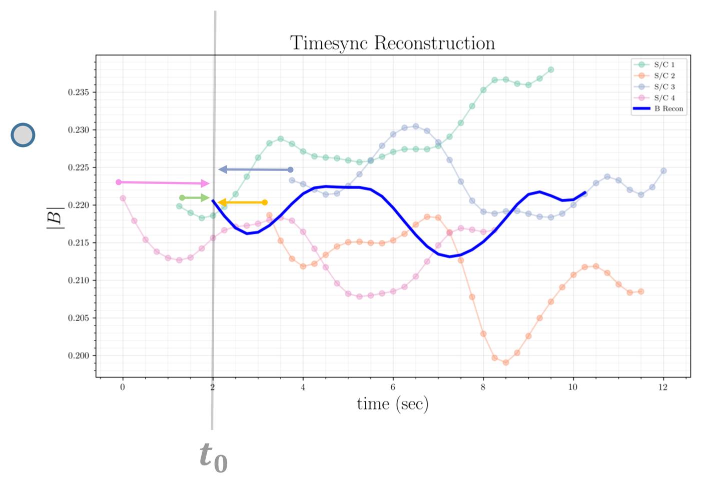
(C) Timesync

The Timesync reconstruction method was created by us using the idea that reconstructing entire timeseries of points in parallel will yield better statistical properties of the reconstructed field when compared to magnetic field reconstructions created point-wise. We do this by first finding the plane that is perpendicular to the bulk flow velocity v_sc. We then reconstructed points on this plane weighted by the (inverse) planar distance to each measurement. Given an optimally chosen weight W, the reconstruction at point r will be given by the expression:

$$B(r) = W\left\{\frac{B\left(r^{(1)}(t_2)\right)}{\left|r^{(1)}(t_2) - r\right|_2^2} + \frac{B\left(r^{(2)}(t_3)\right)}{\left|r^{(2)}(t_3) - r\right|_2^2} + \frac{B\left(r^{(3)}(t_0)\right)}{\left|r^{(3)}(t_0) - r\right|_2^2}\right\}$$

This method can also be viewed from a shifting of timeseries perspective. By finding the time offset of each spacecraft's magnetic field measurements to that of the desired reconstructed timeseries, we are simply shifting each timeseries by a fixed quantity and summing them with an optimal weight.





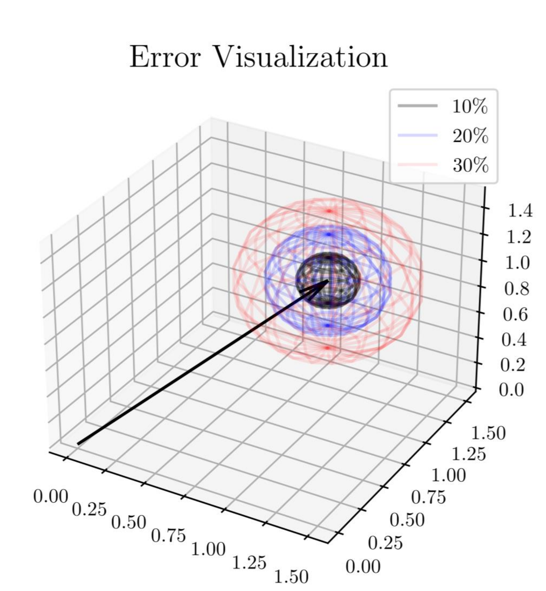
IV. Preliminary Results (Macroscopic)

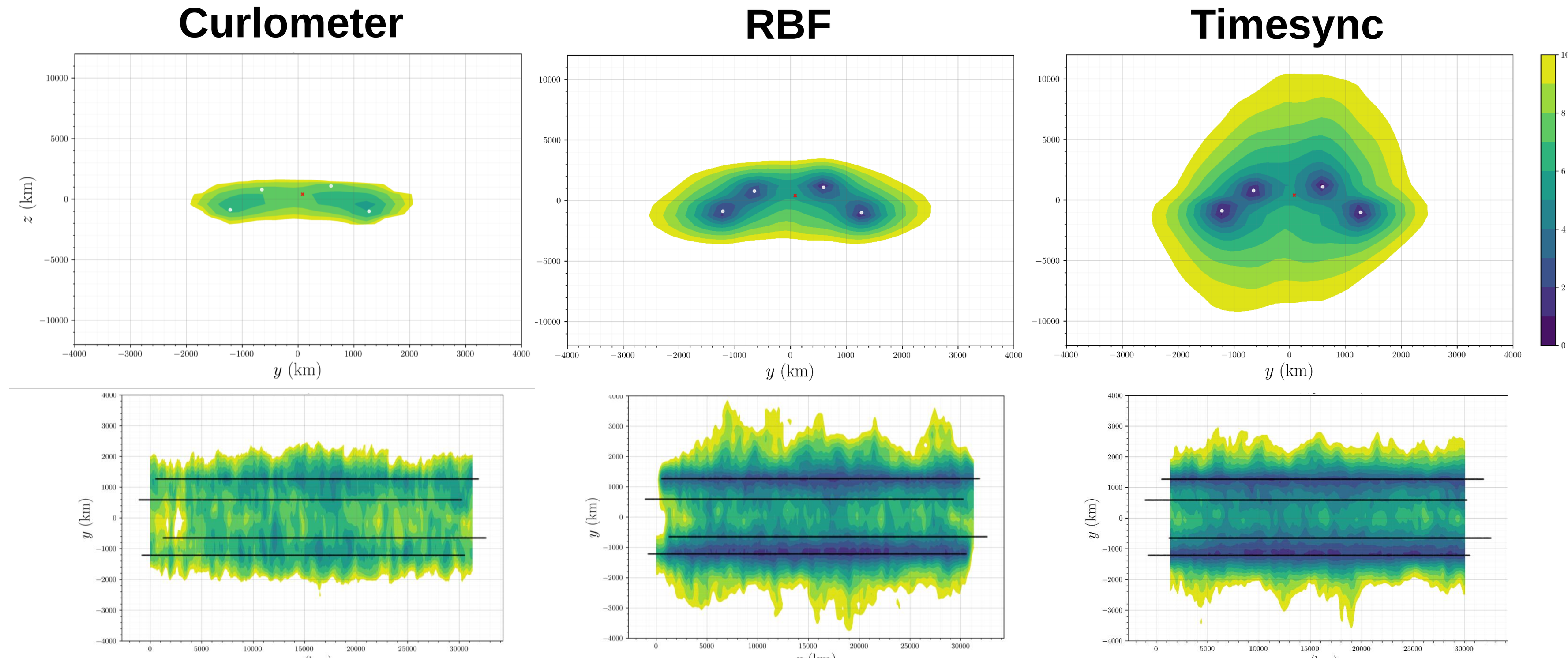
We use our simulated turbulence simulation to generate synthetic timeseries data from a four-spacecraft configuration and implement the three reconstruction methods separately. We then compute the error in the magnetic field at any point in space as

$$\text{Percent Error} = 100\frac{\|B_{calc} - B\|}{\|B\|}.$$

We then plot the average error with respect to the spacecraft configuration/trajectory below. We also compute the percent of the reconstructed magnetic field points which were reconstructed such that they have an error less than 20, 15, 10, 5, and 1%, and display those results in the table. As the figure and table make clear, the Timesync method has the best macroscopic reconstruction and the best computational efficiency.

Volume Fraction	Curlometer	RBF Multiquadric	Timesync
Error < 20%	0.109	0.531	0.917
Error < 15%	0.096	0.382	0.803
Error < 10%	0.062	0.175	0.330
Error < 5%	0.001	0.035	0.043
Error < 1%	0.000	0.001	0.002
Compute Hours	12	1.2	0.75

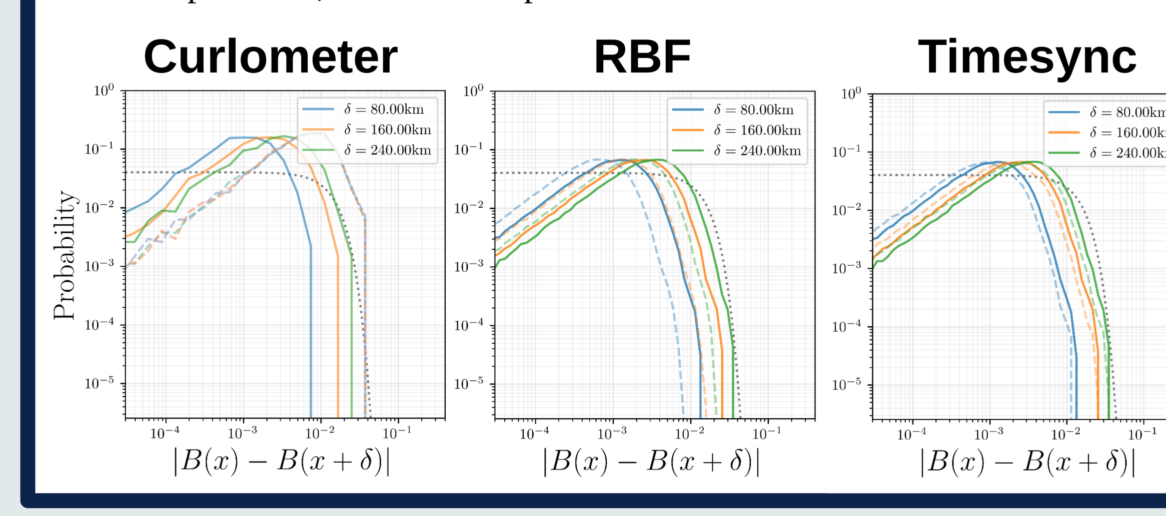




IV. Preliminary Results (Microscopic)

We also analyze the statistical properties of the turbulence simulation and compare them to the statistical properties of the reconstructed magnetic fields resulting from our three reconstruction methods. The results are shown below. The solid lines represent the simulated turbulent magnetic fields and the dashed lines represent the reconstructed magnetic fields. We have included a black dotted line representing a Gaussian distribution for reference.

While none of the methods 'Gaussianize' the distributions, the Curlometer reconstruction clearly performs the worst, as it does not preserve any scale dependent behavior. The RBF reconstruction preserves the shape and relative positions of the distribution of fluctuations; however, these distributions seem to be shifted from the ground truth by a factor of about 2. Finally, the Timesync method appears to preserves the shapes, relative positions, and absolute positions of the distribution of fluctuations.



V. Conclusions/Future Work

From our preliminary simulations it appears that the Timesync reconstruction method is superior to the Curlometer and RBF approaches with respect to reconstructing macroscopic turbulence structures accurately. It also appears that the Timesync method has the added benefit of reconstructing a magnetic field which reproduces the underlying statistics of the actual turbulence it is drawing measurements from.

In our future work, we will repeat this analysis to verify that these results hold for time-evolving turbulent magnetic fields. We also wish to include a fourth reconstruction method in the comparison, the 3D Grad-Shafranov method, as it includes assumptions about the physical properties of the field to assist the reconstruction. Finally, we will perform this analysis for a large bank of randomly generated spacecraft configurations, which contain between 4 and 9 spacecraft each.

Following the data generation and equation learning methodology we developed in Broeren & Klein (2023), we will use the many configurations of spacecraft to get a better understanding of how each of the reconstruction methods accuracy depend on spacecraft configuration. We hope to learn equations that can tell us the expected distribution of errors in the magnetic field reconstruction for an arbitrary spacecraft configuration using each of the reconstruction methods. Our conclusions can be used to select the optimal magnetic field reconstruction technique (and estimate its uncertainty) for future multispacecraft missions measuring space plasma turbulence, such as HelioSwarm.

References

- Broeren, T., Klein, K. G., TenBarge, J. M., et al. Magnetic Field Reconstruction for a Realistic Multi-Point, Multi-Scale Spacecraft Observatory. doi:10.3389/fspas.2021.727076, 2021, FrASS, 8, 144
- Broeren, T., Klein, K. G. Data-driven Uncertainty Quantification of the Wave Telescope Technique: General Equations and Demonstration Using HelioSwarm. doi:10.3847/1538-4365/acc6c7, 2023, ApJS
- Götz Paschmann and Patrick W. Daly. Analysis Methods for Multi-Spacecraft Data. ISSI Scientific Reports Series SR-001, ESA/ISSI, Vol. 1. ISBN 1608-280X, 1998. ISSI Scientific Reports Series, 1, Jan 1998.
- Hakim, A., Loverich, J., and Shumlak, U. (2006). A High Resolution Wave Propagation Scheme for Ideal Two-Fluid Plasma Equations. J. Comput. Phys. 219, 418–442. doi:10.1016/j.jcp.2006.03.036
- Klein, K. G., Spence, H., et al. HelioSwarm: A Multipoint, Multiscale Mission to Characterize Turbulence. arXiv:2306.06537v1, 10, Jun 2023
- Press, WH; Teukolsky, SA; Vetterling, WT; Flannery, BP (2007), "Section 3.7.1. Radial Basis Function Interpolation", Numerical Recipes: The Art of Scientific Computing (3rd ed.), New York: Cambridge University Press, ISBN 978-0-521-88068-8