Path Lengths of Stochastic Parker Field Lines

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SCIENTIFIC MOTIVATION

In the 1960's, Leighton developed a **diffusion** model for the transfer of magnetic flux on the photosphere, and hence, for the turbulent motions of magnetic footpoints on the solar wind source surface.

The Leighton's (1964) model is the basis of the stochastic solar wind magnetic field line model put forth by Jokipii and Parker (1969). However, if assuming a **Brownian diffusion** on the source surface, an infinite path length of magnetic field lines is arrived.

THIS IS DISTURBING.

A SIMPLE SOLUTION

An infinite path length is the consequence of the following property of a diffusion process:

$$< |\Delta r|^2 > \sim \Delta t,$$

at all scales.

One can avoid this "infinite path length problem" by describing the magnetic footpoint motion with a spherical **Ornstein–Uhlenbeck process** with a drift due to the Sun's rotation, on the source surface.

Consequently, the boundary-driven interplanetary magnetic field (IMF) lines become **smooth and differentiable** curves with **finite path lengths** (having a certain probability distribution due to the random nature of the problem).

This model is parameterized by two measurable quantities, the Lagrangian integral timescale and the root-mean-square footpoint velocity. It reduces to Leighton's model in the singular Markov limit when the Lagrangian integral timescale tends to zero while keeping the footpoint diffusivity finite.

Ornstein Uhlenbeck Process

The Ornstein–Uhlenbeck process is defined by the following stochastic differential equation:

$$dx_t = -\theta\, x_t\, dt + \sigma\, dW_t$$

where dW is the Wiener process. An equivalent Langevin equation is,

$$\frac{dx_t}{dt} = -\theta\, x_t + \sigma\, \eta(t)$$

where eta(t) describes the white noise. The Ornstein–Uhlenbeck process is a stationary Gauss–Markov process, and is temporally homogeneous. It describes a random walk process with an "attractive" center, so that particles in the system tend to "fall back" to this center.

The Li+Bian Model (2023)

- Two model parameters: τ_L and V_{rms}^2

$$\frac{d\mathbf{r}_\perp(t)}{dt} = \mathbf{V}_\Omega + \delta\mathbf{V}_\perp(t),$$

$$\frac{d\delta\mathbf{V}_\perp(t)}{dt} = -\frac{\delta\mathbf{V}_\perp(t)}{\tau_L} + \sqrt{\frac{V_{\rm rms}^2}{\tau_L}}\,\boldsymbol{\zeta}(t).$$

These two (fluid parcel) parameters are related to the turbulence characteristics of the MHD plasma at the source surface.

$$\langle \delta\mathbf{V}_\perp(t)\cdot\delta\mathbf{V}_\perp(0)\rangle = V_{\rm rms}^2 e^{-\frac{t}{\tau_L}}. \qquad \lim_{t\to\infty}\int_0^t \langle \delta\mathbf{V}_\perp(0)\cdot\delta\mathbf{V}_\perp(\tau)\rangle\, d\tau = \tau_L V_{\rm rms}^2.$$

The diffusivity κ is given by $\tau_L V_{\rm rms}^2$

Finite Difference Formulation

$$\phi(t+\Delta t) = \phi(t) + \left(\Omega + \frac{\delta V_\phi(t)}{r_0 \sin\theta(t)}\right)\Delta t,$$

$$\theta(t+\Delta t) = \theta(t) + \frac{\delta V_\theta(t)}{r_0}\Delta t,$$

$$\delta V_\theta(t+\Delta t) = \delta V_\theta(t)\left(1-\frac{\Delta t}{\tau_L}\right) + \delta V_\phi(t)\cos\theta(t) \times \left(\Omega + \frac{\delta V_\phi(t)}{r_0\sin\theta(t)}\right)\Delta t + \sqrt{V_{\rm rms}^2\left(\frac{\Delta t}{\tau_L}\right)}\,\zeta_\theta$$

$$\delta V_\phi(t+\Delta t) = \delta V_\phi(t)\left(1-\frac{\Delta t}{\tau}\right) - \delta V_\theta(t)\cos\theta(t) \times \left(\Omega + \frac{\delta V_\phi(t)}{r_0\sin\theta(t)}\right)\Delta t + \sqrt{V_{\rm rms}^2\left(\frac{\Delta t}{\tau_L}\right)}\,\zeta_\phi.$$

Footpoint Trajectory

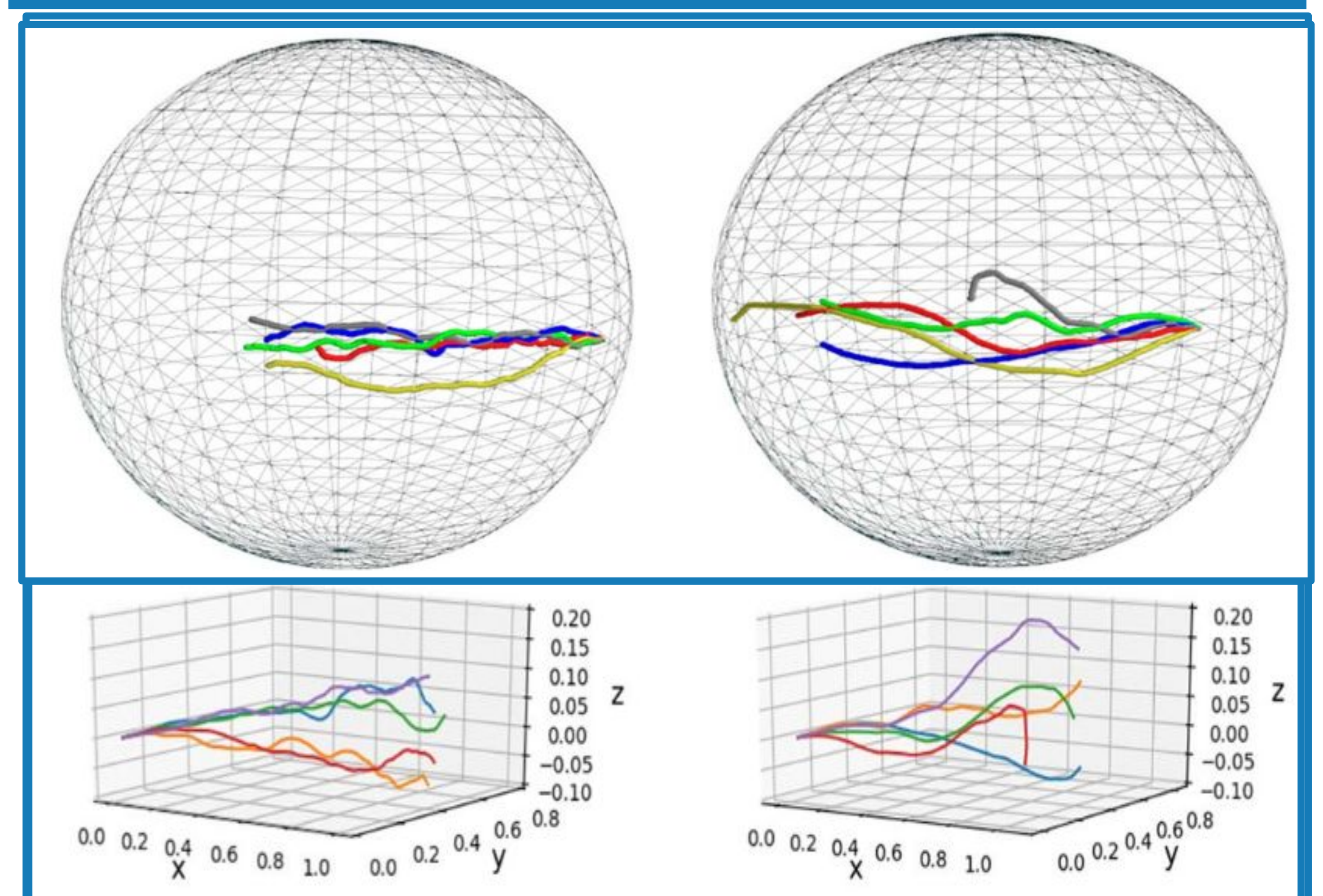

Figure 1: Upper: realizations of footpoint trajectories on the source surface. In both case, Vrms = 1km s^{-1}, while τ_L= 0.2 day in the left and τ_L= 1.0 day in the right panel. Lower Panel: the resulting IMF in 3D. Sun is located at the center. Note the field line from OU process is smooth and differentiable and of finite length. **Diffusion is recovered by keeping diffusivity constant and decreasing τ_L-> 0.**

IMF line path length

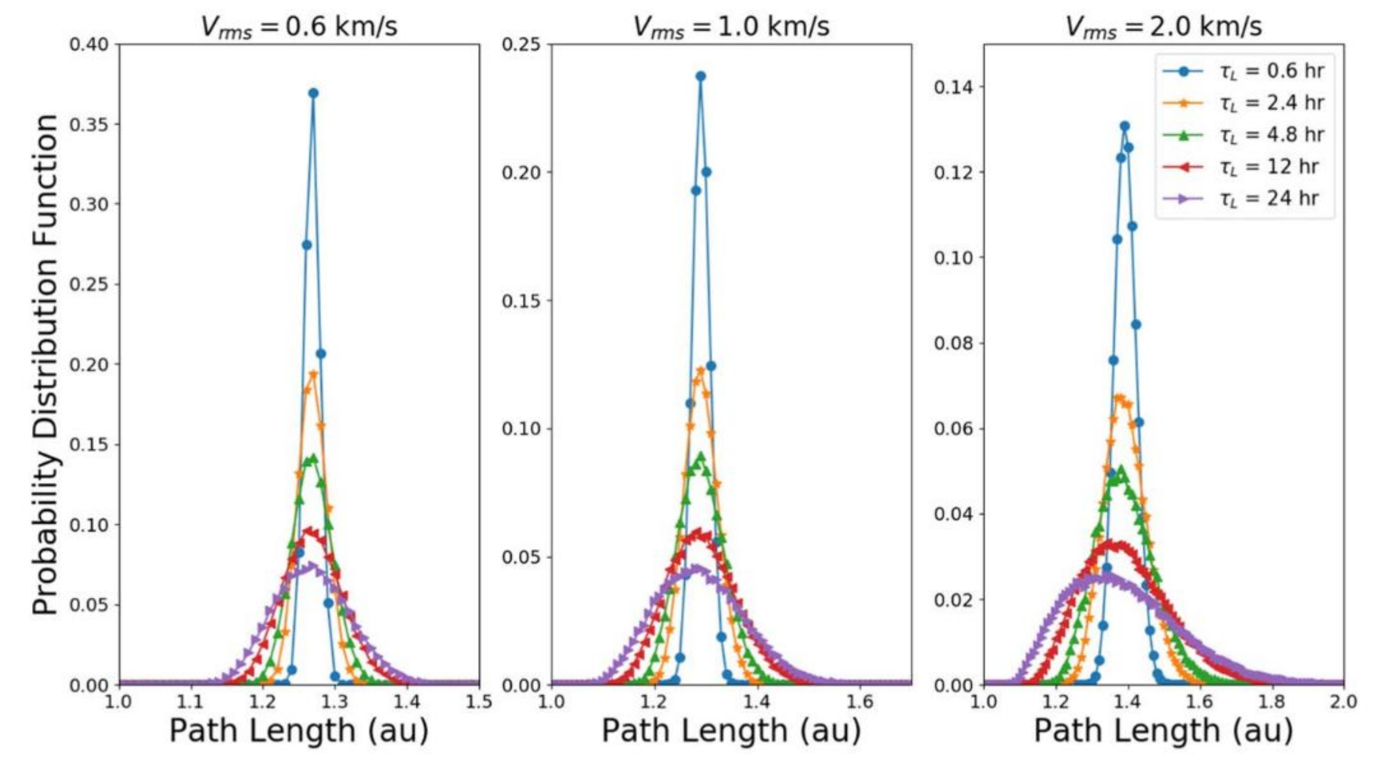

Figure 2: The proboability distribution function P(L, r = 1 au) of the magnetic path length L at 1 au. The left panel corresponds to Vrms = 0.6 km s^{-1} at the middle panel to Vrms = 1.0 km s^{-1}, and the right panel to V rms = 2.0 km s^{-1}. Path length is computed through,

$$L(r) = \int_0^{r/V_{sw}} dt\,\sqrt{V_{sw}^2 + r^2(t)\left(\frac{d\theta(t)}{dt}\right)^2 + r^2(t)\sin^2\theta(t)\left(\frac{d\phi(t)}{dt}\right)^2}$$

Relating to observations

- One can use observed electron path length from FVDA to constrain Vrms, e.g., Zhao et al. (2019).
- Li+Bian (2023) only considered turbulence contribution at source surface, but ignored contribution from in-situ turbulence, which is r-dependent and is examined in Bian+Li (2022).
- A fully heliospheric stochastic IMF can be constructed with a frozen-in condition and piecewise OU process with local turbulence description.
- Propagation of energetic particles on these stochastic IMFs can be compared with multi-s/c observations, including Q/A-dependent spreading.

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