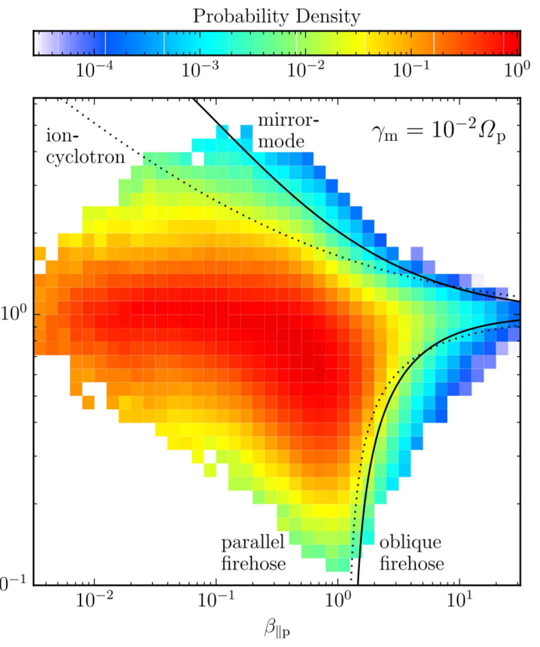
Beyond Brazil: Tools for Assessing the Impacts of non-Maxwellian Structures on Plasma Behavior

Kristopher G Klein[1], Daniel Verscharen[2], Jada Walters[1], Mihailo Martinovic[1], Emily Lichko[3], Michael Stevens[4], Benjamin Chandran[5]
1) University of Arizona; 2) Mullard Space Science Laboratory; 3) University of Chicago; 4) Harvard Smithsonian Center for Astrophysics; 5) University of New Hampshire

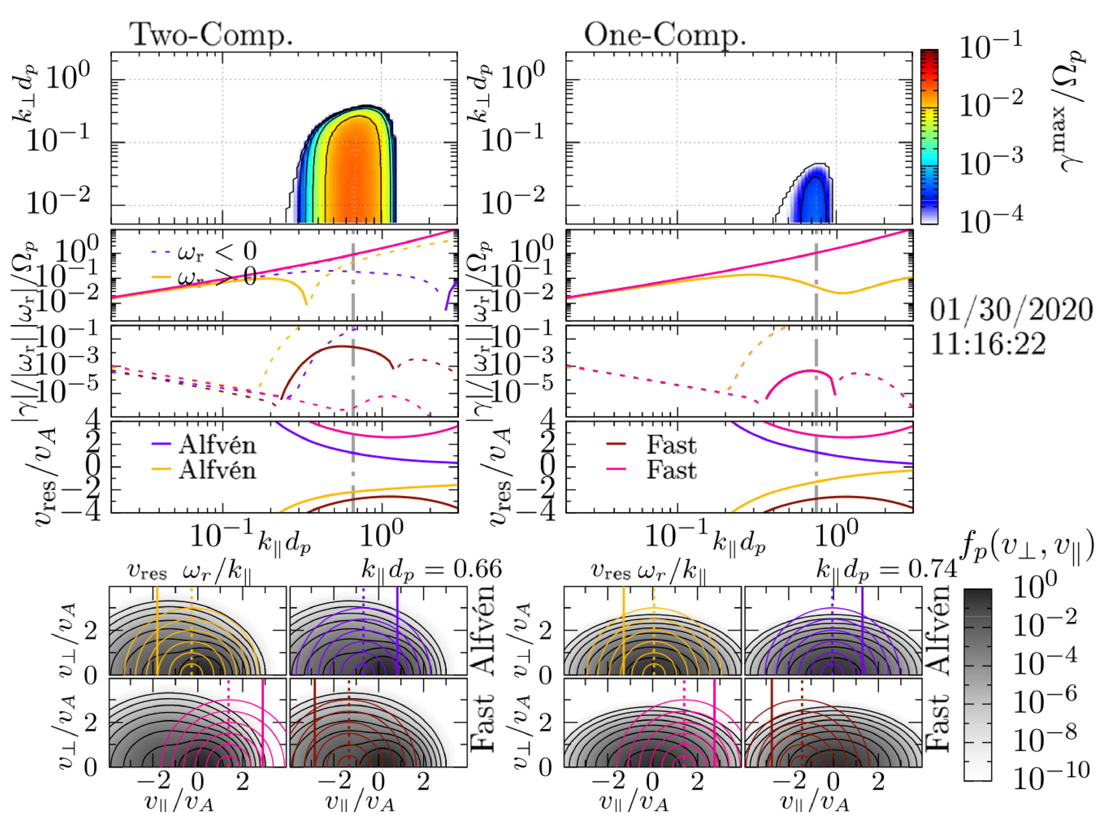
Instability Analysis with Multiple Ions Components using Plasma in a Linear Uniform Magnetized Environment (PLUME)



Verscharen et al 2019 LRSP (Wind)

The Solar Wind is a weakly collisional plasma, with low densities and high temperatures enabling non-isotropic Maxwellian features in the velocity distribution (VDF) to evolve and persist. One obvious feature is the *pressure anisotropy*. Departures from isotropic pressures are known to drive linear instabilities, canonically illustrated in *Brazil plots.*

However, other non-Maxwellian structures can alter the linear plasma plasma response. Consider an example case where the proton VDF is modeled as either a single population or relatively drifting core-and-beam. Due to the different phase-space densities resonantly interacting with the wave, the frequency and growth rates are significantly altered.



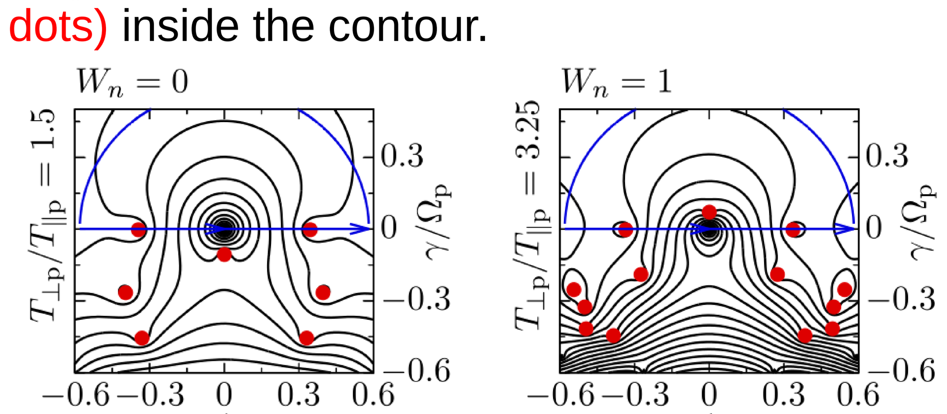
Klein et al 2021 ApJ (PSP)

We want to accurately include the *bumps and wiggles* of the VDFs into our calculation of the linear response. Our first tool to do this is the PLUME dispersion solver, which reads a set of dimensionless parameters *P*

$$\mathcal{P}_R = \left\{\beta_{\parallel,R}, \frac{v_{t\parallel,R}}{c}\right\} \quad \mathcal{P}_j = \left\{\frac{n_j}{n_R}, \frac{T_{\parallel,R}}{T_{\parallel,j}}, \frac{T_{\perp,j}}{T_{\parallel,j}}, \frac{\Delta v_{j,R}}{v_{A,R}}, \frac{m_j}{m_R}, \frac{q_j}{q_R}\right\}$$

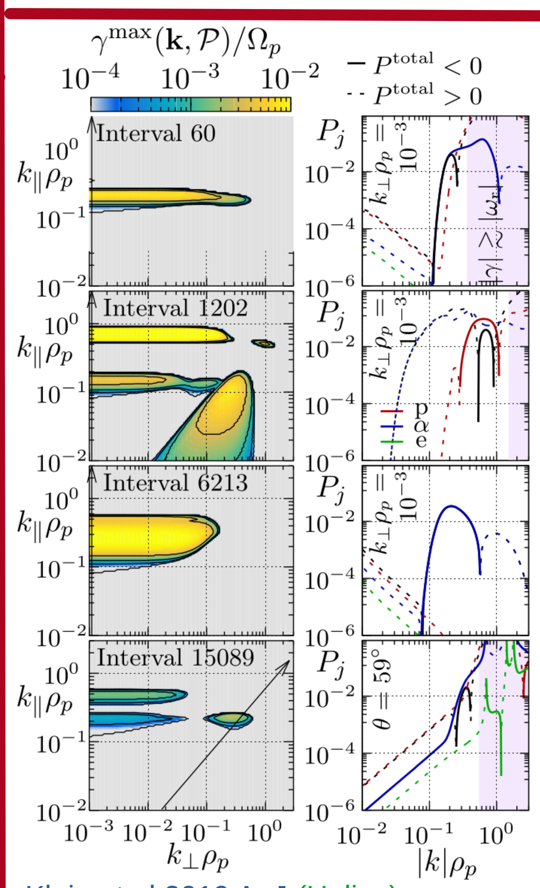
for an arbitrary number of relatively drifting bi-Maxwellians (*j*) and solves the dispersion relations ω(k) that satisfy the wave equation |D(ω(k))|=0, following Chapter 10 of Stix 1992.

To the fidelity that the underlying VDF can be represented with a collection of bi-Maxwellians, this provides the full linear solution and associated eigenfunctions. If we want to investigate the stability of an equilibrium, we can implement **Nyquist's criterion** and perform a contour integral of 1/|D(ω(k))| over the upper half complex frequency plane (blue curves). If the system is unstable, the integral will be non-zero, as there will be solutions (red dots) inside the contour.



This integration was implemented in Klein et al 2017 JGR using the PLUME dispersion solver allowing the automated evaluation of linear stability.

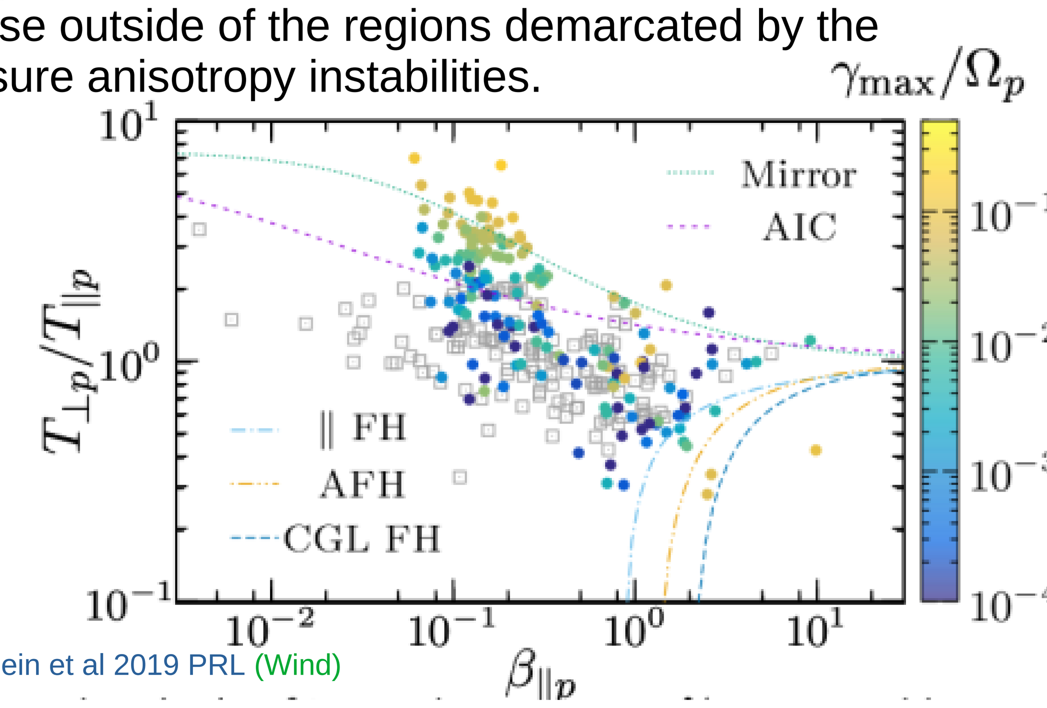
Statistical Variations in Instability Behavior with PLUME



Klein et al 2019 ApJ (Helios)

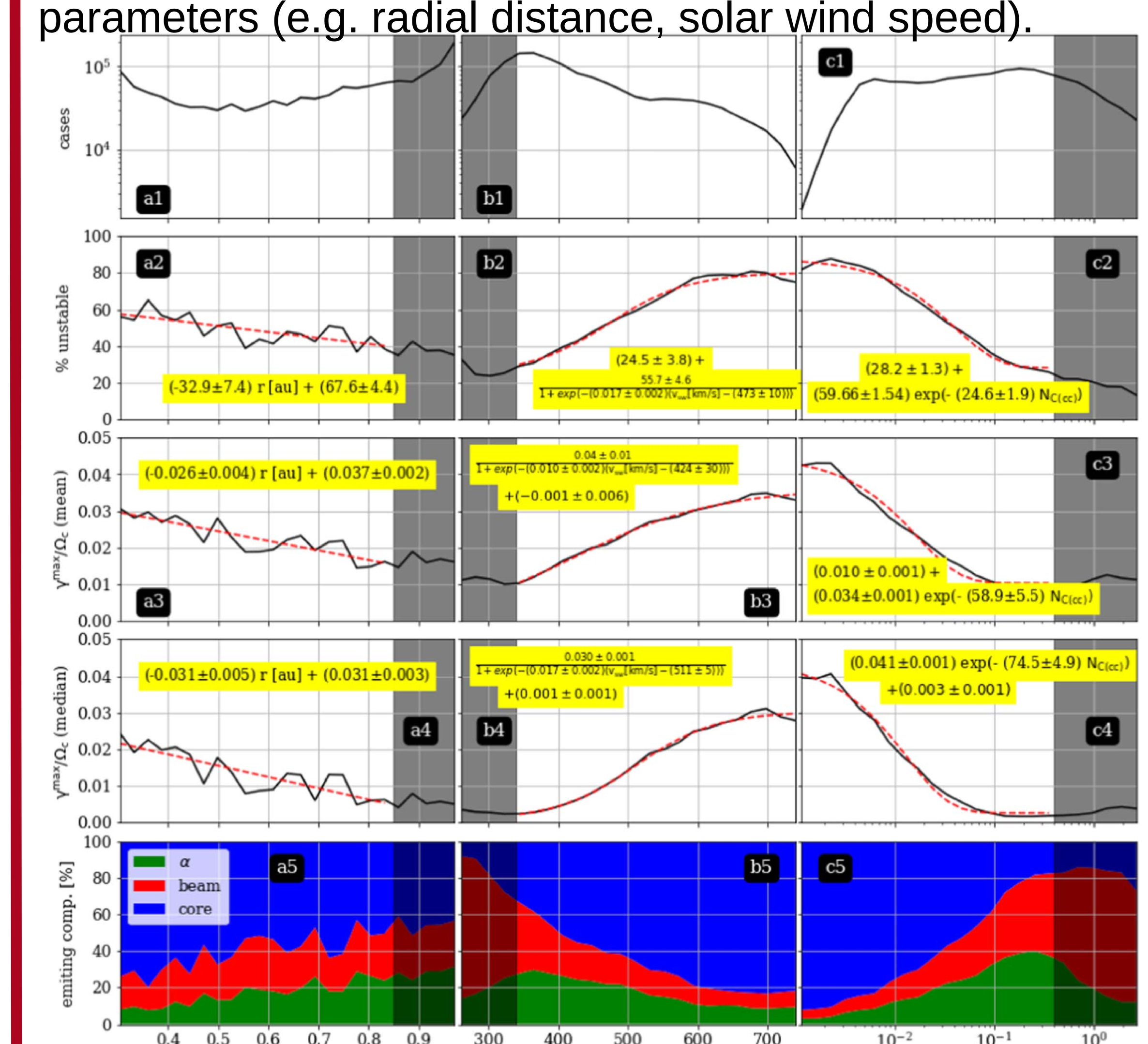
Using Nyquist's criterion enables the study of instability properties across a wide range of VDF parameters and wavevectors, and will identify when several different instabilities can be driven by different sources of free energy (e.g. by both the proton anisotropy and relative alpha drift, fourth row) **or** when a secondary population acts to reduce the growth rate of an unstable mode (e.g. second row).

Using this method, we see that instabilities are predicted to arise outside of the regions demarcated by the pressure anisotropy instabilities.



Klein et al 2019 PRL (Wind)

Applying this method to ~1.5 million proton core, proton beam, and alpha VDF fits from the Helios spacecraft (Durovcová et al 2019 Solar Physics), we are able to resolve how the changes in the ion VDFs lead to changes in the predicted instabilities as a function of solar wind parameters (e.g. radial distance, solar wind speed).



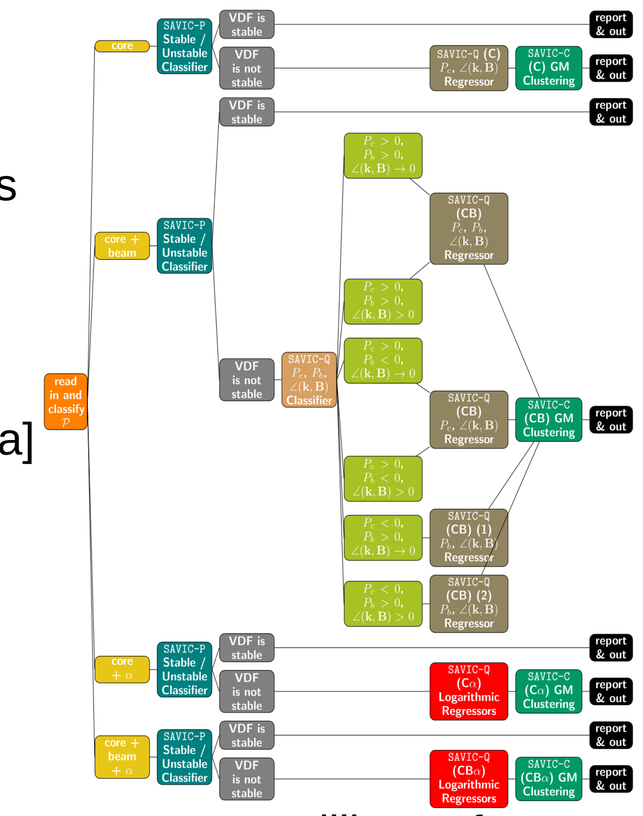
Martinovic et al 2021 ApJ (Helios)

PLUME is being converted into an open source code that will be soon released publicly. A selected bibliography of papers using PLUME for instability analysis:
Klein et al 2017 JGR, Klein et al 2018 PRL, Klein et al 2019 ApJ, Verniero et al 2020 ApJS, Vech et al 2021 A&A Klein et al 2021 ApJ, Martinovic et al 2021 ApJ, Verniero et al 2022 ApJ. See also JET-PLUME Poster by C. Brown.

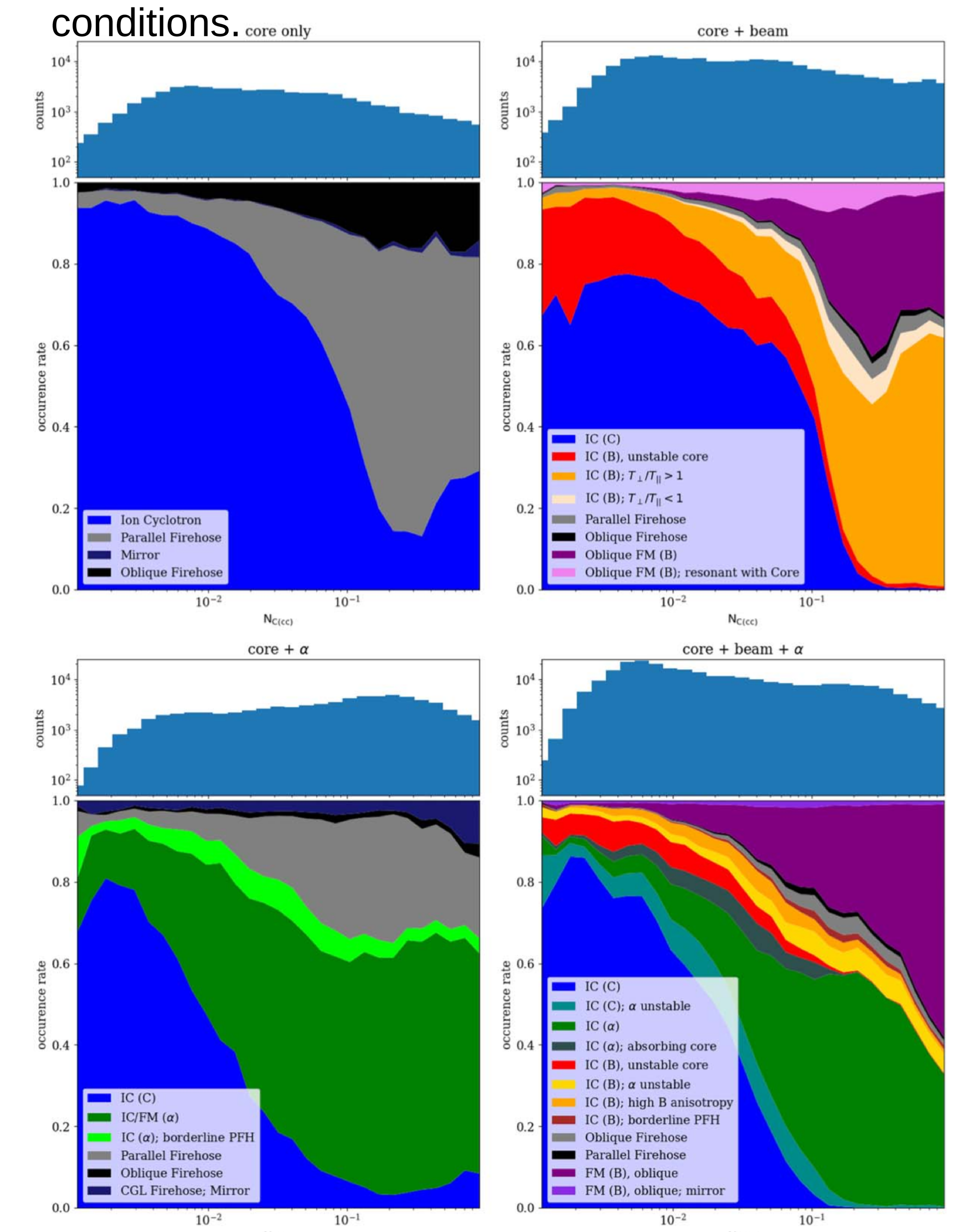
Machine Learning Instability Analysis with Stability Analysis Vitalizing Instability Classification (SAVIC)

A fundamental limitation of the Nyquist method is its speed, especially in processing statistically large datasets. To overcome this obstacle, we have trained a series of machine learning algorithms that, given the set of parameters *P*, can predict if an equilibrium is unstable (SAVIC-P), quantify the characteristics of the unstable wave using a regressor (SAVIC-Q), and then classify the unstable wave using clustering (SAVIC-C).

At present, SAVIC has been trained instability solutions from four subsets of data:
-proton core,
-proton core +beam,
-proton core +alpha,
-proton core +beam +alpha]
as processed by PLUME, and is able to accurately predict and classify the unstable modes at rates greater than 96% for the most complex cases.



This ML method allows us to process millions of intervals in seconds on a desktop computer, and to immediately classify the kinds of instabilities that are arising, allowing for much more detailed statistical studies for which kinds of instabilities arise under what conditions.

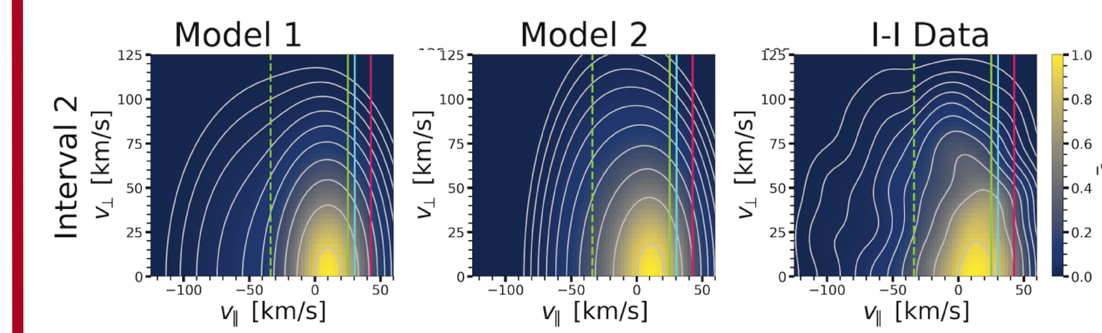


Martinovic & Klein 2023 ApJ (Helios)

The SAVIC Code is described in Martinovic & Klein 2023 ApJ, trained using the Helios Ion VDF fits from Durovcová et al 2019 Solar Physics. The python package is available via pip install, with details and jupyter notebook tutorials available at https://savic.readthedocs.io

The Arbitrary Linear Plasma Solver

Actual Solar Wind VDFs are frequently not well described by even a collection of bi-Maxwellians.

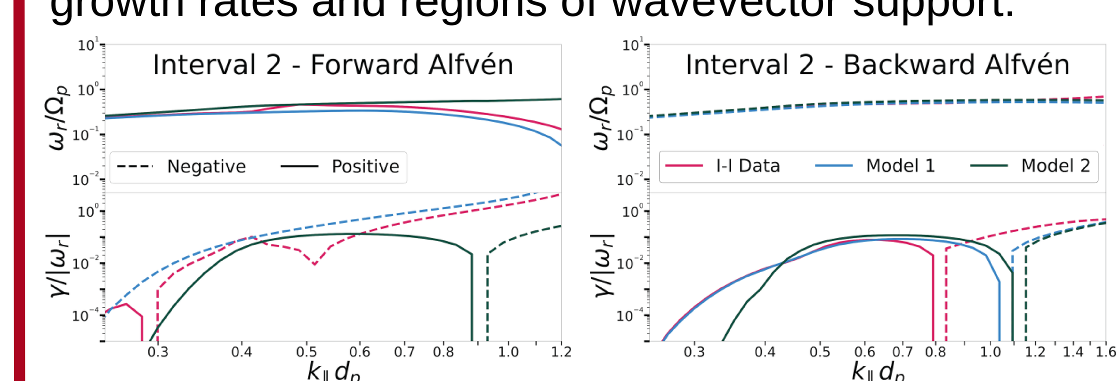


Walters et al 2023 ApJ [under review] (Wind; Model 1&2 core+beam proton fits to Faraday Cup data; I-I Data represents a statistically more likely description of the measurements given the known instrument response.)

In such cases, we need to numerically integrate the VDF f_j for a more accurate plasma susceptibility χ_j, rather than rely on closed form solutions for analytic functions.

ALPS implements this integration with a parallelized scheme and takes as inputs gyrotropic grids for f_j in parallel and perpendicular momentum space, with analytic continuation handled with a hybrid scheme. ALPS works for both non-relativistic and relativistic plasmas, and can treat individual components as bi-Maxwellians to decrease computational costs.

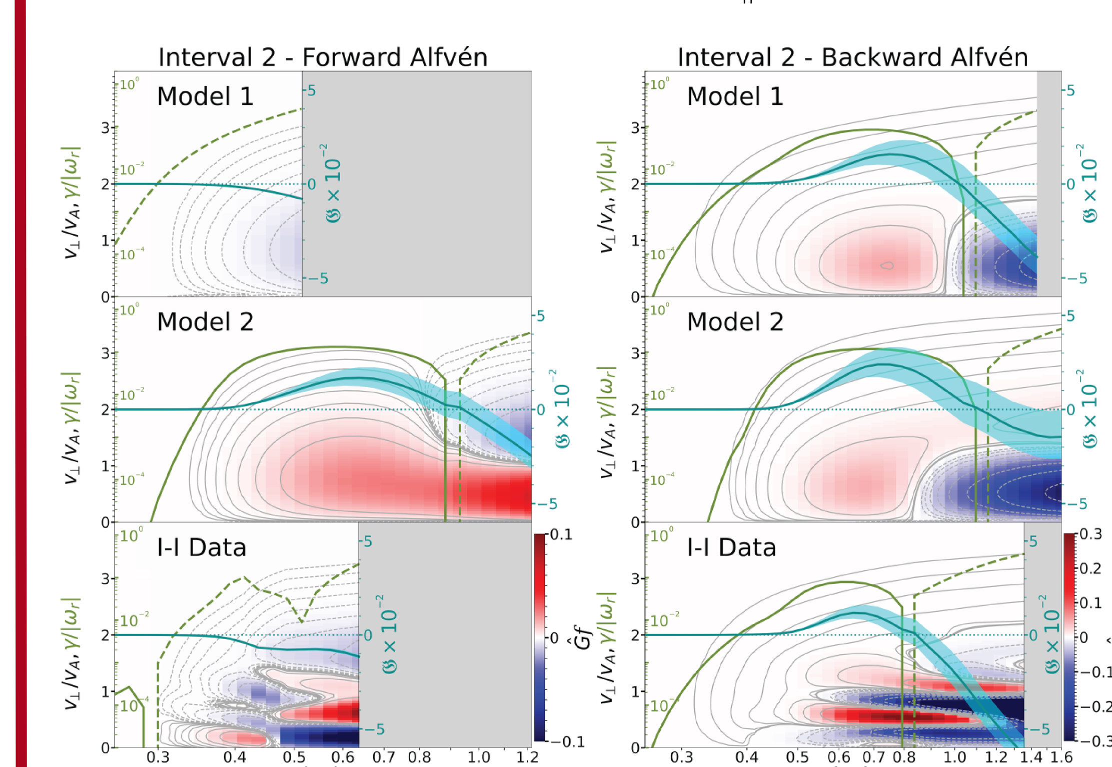
Comparing dispersion relations for intervals predicted to be unstable for bi-Maxwellian core+beam proton distributions, we find significant changes to the growth rates and regions of wavevector support:



Walters et al 2023 ApJ [under review] (Wind)

We can understand the difference by calculating the quasilinear diffusion as a function of scale and velocity space, identifying what structures of the distribution function which are not captured by the bi-Maxwellian model are responsible for absorbing energy or emitting waves.

$$\frac{\gamma_j}{\omega_r} \propto \left[\left(1 - \frac{k_\parallel v_{res}}{\omega_r}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega_r}\frac{\partial}{\partial v_\parallel}\right] f_j[v_{res}(k_\parallel d_p), v_\perp]$$



Walters et al 2023 ApJ [under review] (Wind)

The ALPS Code is described in Verscharen et al 2018 JPP and is publicly available on github at danielver02.github.io/ALPS/
The FORTRAN90 code is open source and introductory tutorials are included on the website.