# Electron heating at Earth's quasi-perpendicular bow shock measured by MMS: a relative comparison of compression and magnetic pumping

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## Shock acceleration requires seed population

Particle acceleration at shocks is widely accepted to be driven by diffusive shock acceleration (DSA), which requires a seed population of particles pre-accelerated to moderately high energies.

Acceleration of this seed population, known as the "injection problem", is particularly difficult for electrons that need high frequency waves to scatter.

Magnetic pumping is a viable acceleration/heating mechanism for producing a seed population of electrons.

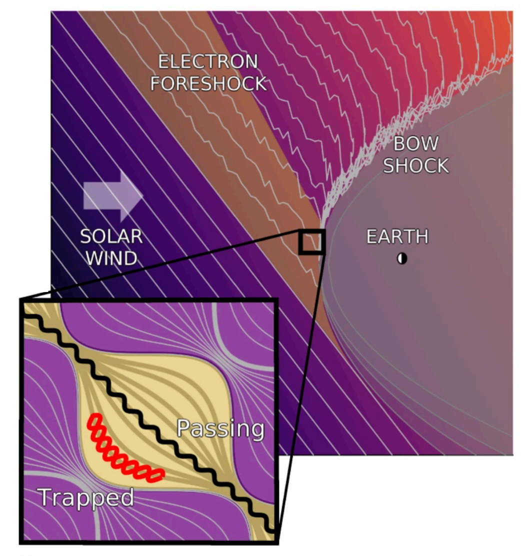
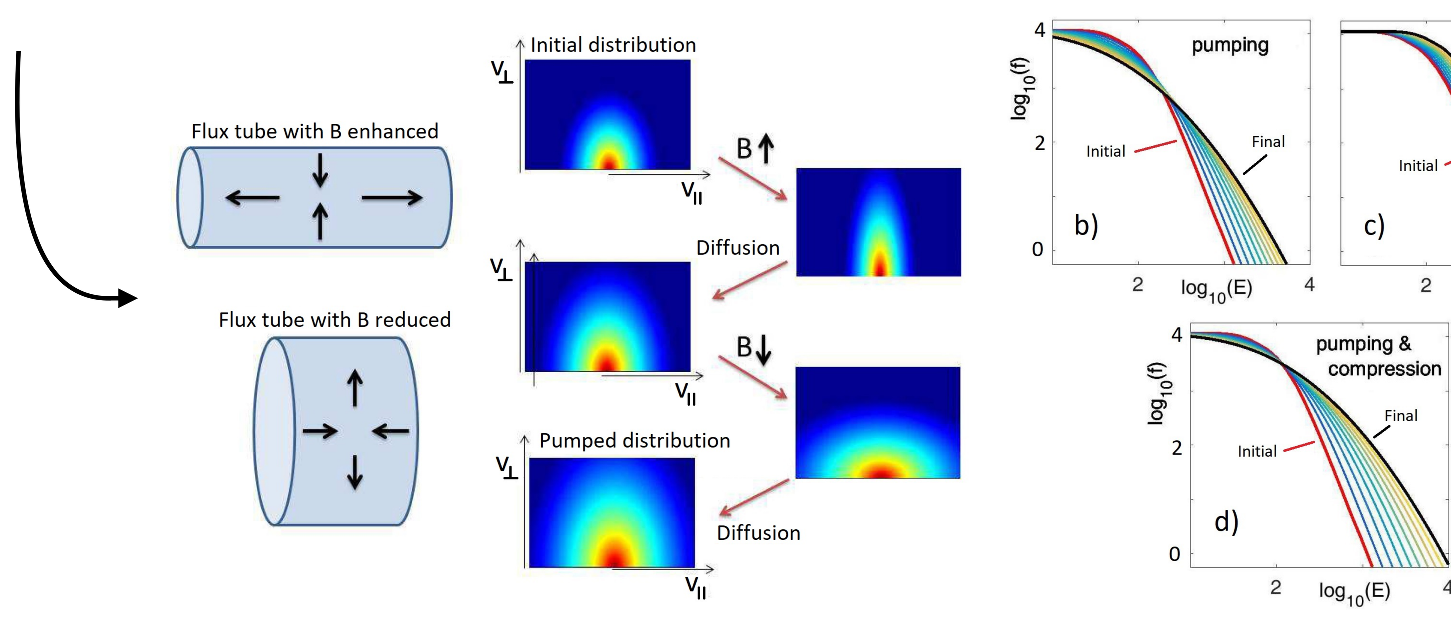



Illustration of passing and trapped particles in compressional magnetic fluctuations upstream of Earth's bow shock

## Magnetic pumping heats particles

For particles with conserved adiabatic invariants $\mu = mv_\perp^2/(2B)$ & $j = \oint mv_\parallel d\ell$, pressures parallel and perpendicular to the magnetic field follow the CGL $\longrightarrow$ $\begin{cases} p_\perp \sim nB \\ p_\parallel \sim n^3/B^2 \end{cases}$

Under the influence of cyclic **magnetic perturbations** and **particle scattering** that induces pitch angle diffusion, non-adiabatic energization of particles occurs, termed **magnetic pumping**.




We model this process by separating the distribution function in velocity space into the parallel passing (+), antiparallel passing (-), and trapped populations (T):

Krook-like pitch angle scattering; $R_\parallel$ is fraction of phase space volume of passing; $\lambda_s$ is electron mean free path

s parameterizes distance along shock

adiabatic compression (limit of high scattering)

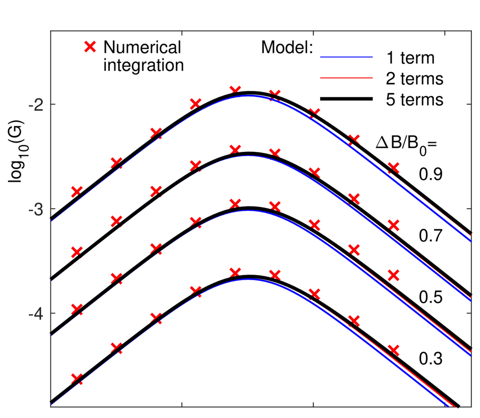
$$\frac{\partial f_{+\parallel}}{\partial s} = \frac{1-R_\parallel}{\lambda_s}\left(f_T - f_{+\parallel}\right)$$

$$\frac{\partial f_{-\parallel}}{\partial s} = \frac{1-R_\parallel}{\lambda_s}\left(f_{-\parallel} - f_T\right)$$

$$\frac{\partial f_T}{\partial s} = -\frac{1}{3}\frac{\partial \log B}{\partial s}\frac{\partial f_T}{\partial \log v} + \frac{\omega_{\text{pump}}}{u_{sh}}\mathcal{G}\frac{1}{v^3}\frac{\partial}{\partial \log v}\left(v^3\frac{\partial f_T}{\partial \log v}\right) + \frac{R_\parallel}{\lambda_s}\left(f_{+\parallel} - f_{-\parallel}\right)$$

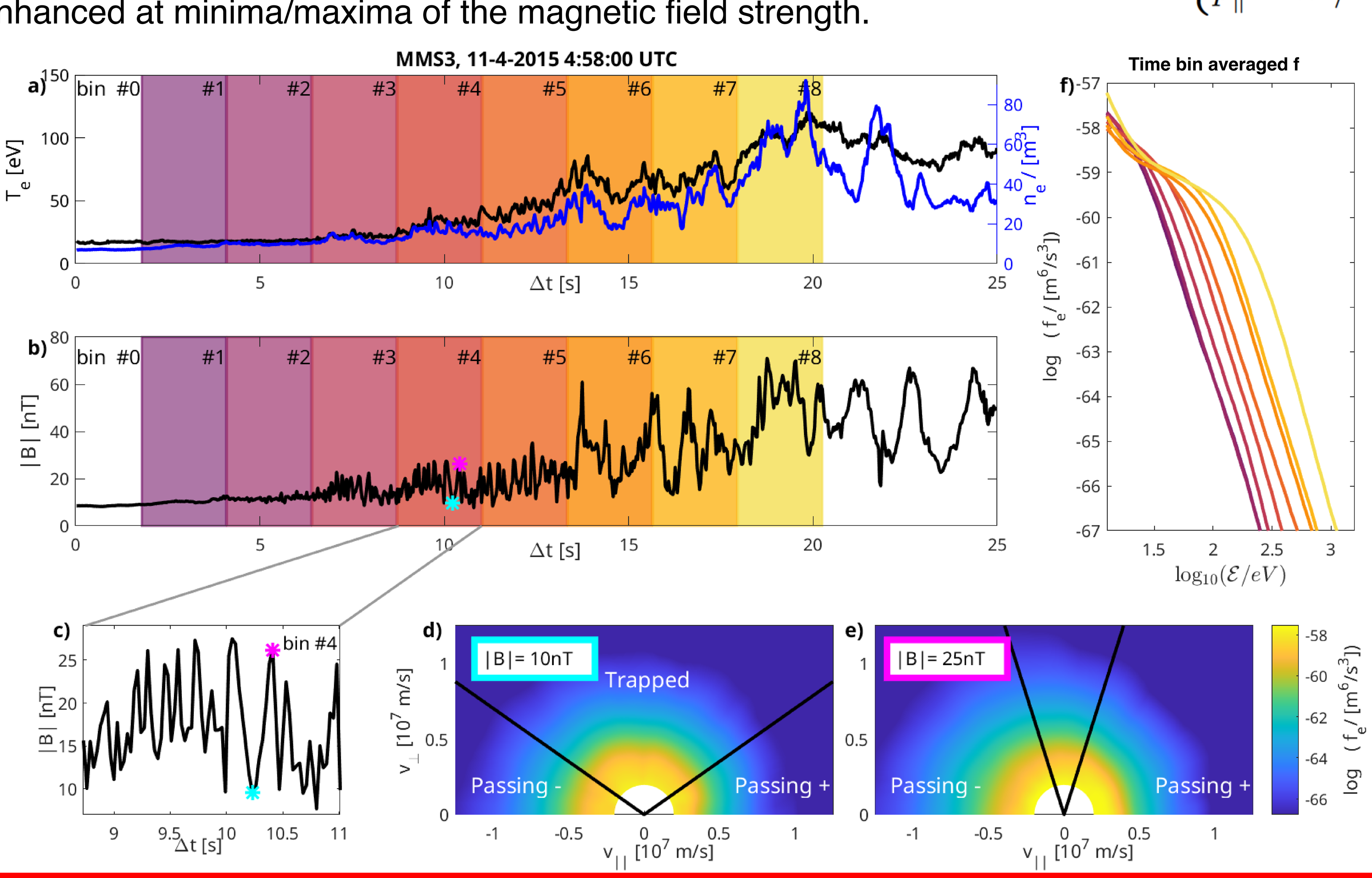
The slow evolution of $f$ from magnetic pumping is modeled as speed diffusion in velocity space **only acting on trapped particles**, with an efficiency per pump cycle, $\mathcal{G}$, that depends on the ratio of scattering to pump frequency and strength of magnetic fluctuations.

$$\mathcal{G}\left(\frac{\nu C_K}{\omega}\right) \equiv \frac{1}{4}\sum_n \frac{\nu C_K/\omega}{n^2 + (\nu C_K/\omega)^2}\left\langle (g^n h^n)_\Lambda \right\rangle_t \simeq \frac{0.02\,\nu/\omega}{4+\nu^2/\omega^2}\left(\left(\frac{\delta B}{B}\right)^{2.6} + 3\left(\frac{\delta B}{B}\right)^{5.6}\right)$$



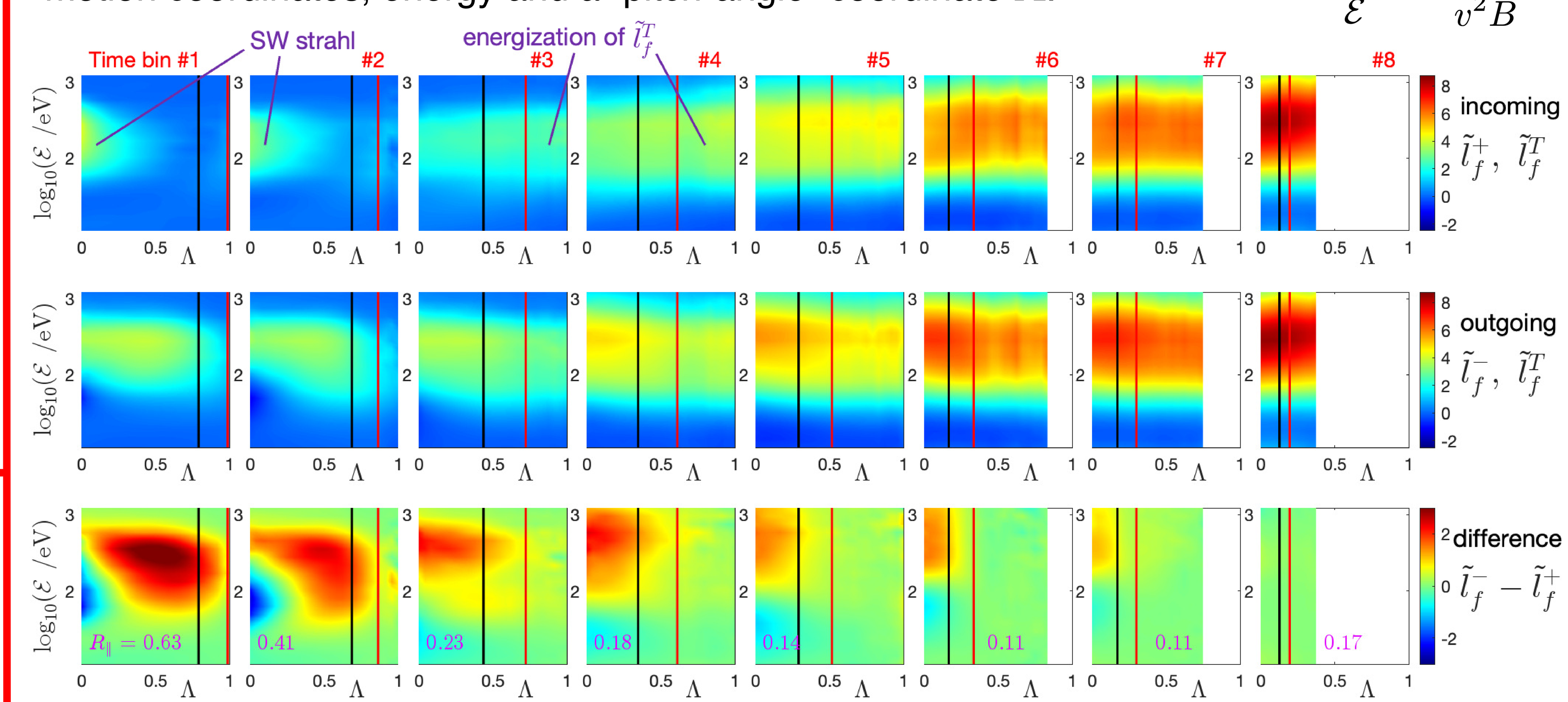

## CGL-like behavior observed in bow shock $\delta B$ fluctuations

Below is MMS3 data from a quasi-perpendicular shock crossing with shock normal angle $\theta_{Bn} \sim 84°$. Distributions in foreshock are parallel/perpendicularly enhanced at minima/maxima of the magnetic field strength. $\begin{cases} p_\perp \sim nB \\ p_\parallel \sim n^3/B^2 \end{cases}$




## Pumping explains MMS bow shock data

For evaluation of TP-boundaries and evaluation of heating as a function of energy, it is convenient to switch to constant of motion coordinates, energy and a "pitch-angle" coordinate $\Lambda$. $\longrightarrow$ $\mathcal{E} = \frac{mv^2}{2}$, $\Lambda = \frac{\mu B_0}{\mathcal{E}} = \frac{v_\perp^2 B_0}{v^2 B}$



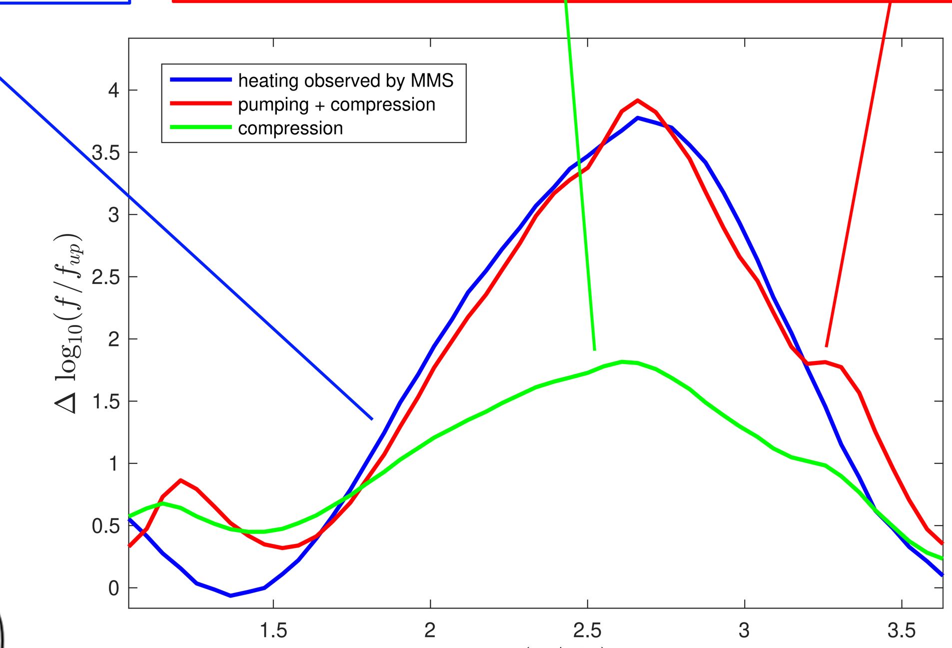

$\tilde{l}_f^\alpha \equiv \log(f_\alpha/\langle f_{+\text{up}}\rangle)$

The model equations can be recast into log form and integrated in distance along the shock, s, to more easily apply model to spacecraft data.

$$[\tilde{l}_f^T]_{s_3}^{s_8} + \left\langle \frac{R_\parallel}{1-R_\parallel}\right\rangle_s [\tilde{l}_f^+ + \tilde{l}_f^-]_{s_3}^{s_8} \simeq -\frac{1}{3}[l_B]_{s_3}^{s_8}\left\langle H_{\text{comp}}(l_f^T)\right\rangle_s + \left\langle H_{\text{pump}}(l_f^T)\right\rangle_s 2\pi N_p\mathcal{G}$$

We directly evaluate the **total change in log $f$ (blue)** and the contribution from **compression (green)** from the data, while the effect of **magnetic pumping (red)** is optimized by the scalar factor $2\pi N_p \mathcal{G}$ and added to compression to fit the blue curve.

$$\begin{cases} H_{\text{comp}}(l_f) = 2\frac{\partial l_f}{\partial l_\mathcal{E}} \\ H_{\text{pump}}(l_f) = 4\left(\frac{3}{2}\frac{l_f}{l_\mathcal{E}} + \left(\frac{\partial l_f}{\partial l_\mathcal{E}}\right)^2 + \frac{\partial^2 l_f}{\partial l_\mathcal{E}^2}\right) \end{cases}$$




Additionally, the log-form of the passing equations can be rearranged to get an expression for the shock-width normalized **electron mean free path**.

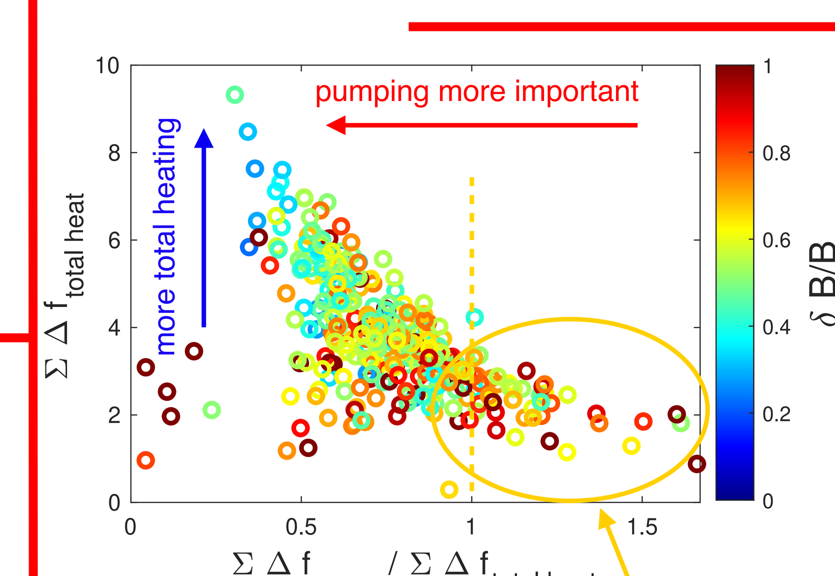
$$\frac{\partial \tilde{l}_f^\pm}{\partial s} \simeq \pm\frac{1-R_\parallel}{\lambda_s}\left(\exp\left[\pm(\tilde{l}_f^- - \tilde{l}_f^+)/2 - 1\right]\right)$$

**For this event, $\lambda_s/(s_8 - s_3) \sim 0.14$**

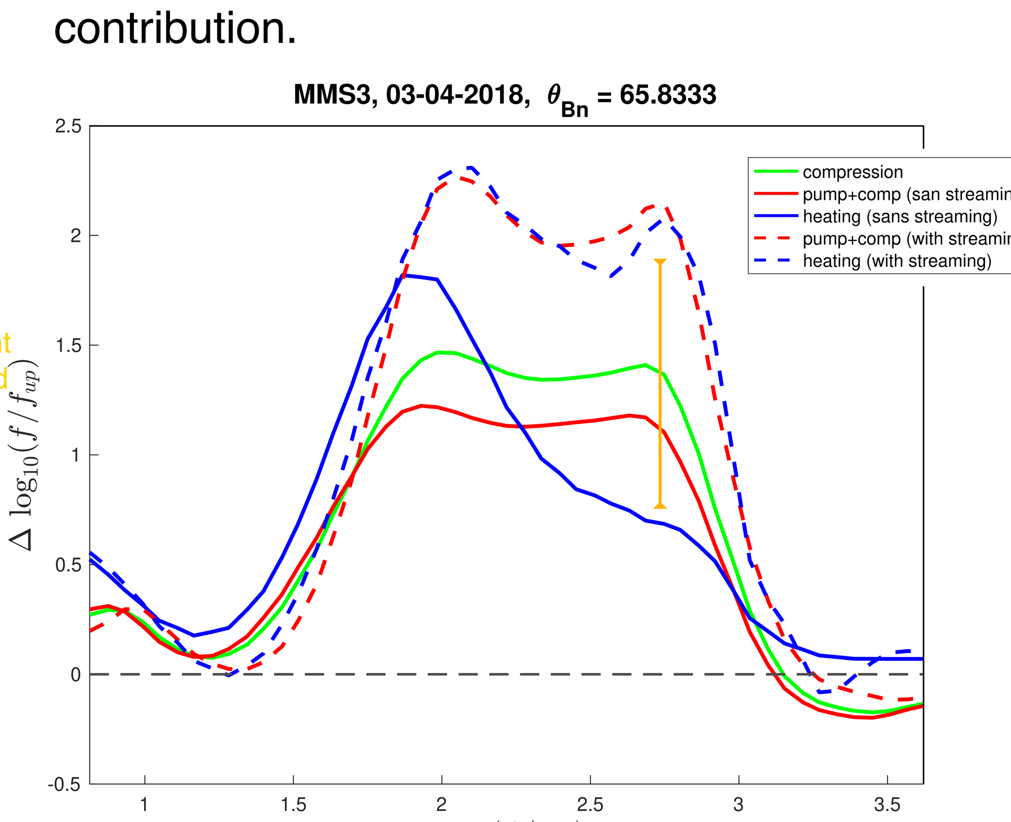
add (+) and (-); integrate $\longrightarrow$

$$\frac{\lambda_s}{s_8 - s_3} \simeq \frac{1-R_\parallel}{[\tilde{l}_f^+ + \tilde{l}_f^-]_{s_3}^{s_8}}\left\langle \exp\left((\tilde{l}_f^- - \tilde{l}_f^+)/2\right) - \exp\left((\tilde{l}_f^+ - \tilde{l}_f^-)/2\right)\right\rangle_s$$

## Thermal streaming improves model




In many events (taken from database compiled by Lalti et. al. 2022), the contribution from compression overtakes the total heating contribution.




MM3 on January 3, 2020 over a shock crossing with $\theta_{Bn} \sim 65°$. Here, the dashed red (pumping) and dashed green (compression) include effects of thermal streaming.

Since compression term is the limit of high scattering, we add velocity advection terms from Vlasov equation to capture physics of thermal streaming in our model.

$$\frac{\partial f_{+\parallel}}{\partial s} = \frac{1-R_\parallel}{\lambda_s}\left(f_T - f_{+\parallel}\right) - \frac{1}{u_{sh}}\nabla\cdot(\mathbf{v}f_{+\parallel})$$

$$\frac{\partial f_{-\parallel}}{\partial s} = \frac{1-R_\parallel}{\lambda_s}\left(f_{-\parallel} - f_T\right) - \frac{1}{u_{sh}}\nabla\cdot(\mathbf{v}f_{-\parallel})$$

Spatial advection of passing particles (thermal streaming) contributes to total heating contribution

$$[\tilde{l}_f^T]_{s_i}^{s_f} + \left\langle \frac{R_\parallel}{1-R_\parallel}\right\rangle_s \left\{[\tilde{l}_f^+ + \tilde{l}_f^-]_{s_i}^{s_f} + \frac{\langle B\rangle_s}{u_{sh}}\left(\frac{1}{\left\langle e^{\tilde{l}_f^+}\right\rangle_s}\left[\frac{|v_\parallel| e^{\tilde{l}_f^+}}{B}\right]_{s_i}^{s_f} + \frac{1}{\left\langle e^{\tilde{l}_f^-}\right\rangle_s}\left[\frac{-|v_\parallel| e^{\tilde{l}_f^-}}{B}\right]_{s_i}^{s_f}\right)\right\} \simeq$$

$$-\frac{1}{3}[l_B]_{s_i}^{s_f}\left\langle H_{\text{comp}}(l_f^T)\right\rangle_s + \left\langle H_{\text{pump}}(l_f^T)\right\rangle_s 2\pi N_p\mathcal{G}$$

assume trapped particles have zero spatial divergence for advection; apply divergence theorem

**References:**

- Egedal, J., & Lichko, E. (2021). The fast transit-time limit of magnetic pumping with trapped electrons. *Journal of Plasma Physics, 87*(6), 905870610. doi:10.1017/S0022377821001173
- Lalti, A., et. al. (2022). A database of MMS bow shock crossings compiled using machine learning. *Journal of Geophysical Research: Space Physics*, 127, e2022JA030454. https://doi.org/10.1029/2022JA030454

**Acknowledgements:**
This work is based upon work supported by NASA under award 80NSSC23K0088.