HelioCubed: A 4^{th}-Order Accurate MHD Solver for Solar Wind and CME Modeling on a Cubed-Sphere Grid

Talwinder Singh[1], Phillip Colella[2,3], Brian Van-Straalen[2], Chris Bozhart[2], Nikolai Pogorelov[4,5]

[1]Department of Physics & Astronomy, Georgia State University, GA, USA
[2]Applied Numerical Algorithms Group, Lawrence Berkeley National Laboratory, Berkeley, CA, USA
[3]Electrical Engineering and Computer Science Department, University of California Berkeley, Berkeley, CA, USA
[4]Department of Space Science, University of Alabama in Huntsville, AL, USA
[5]Center for Space Plasma and Aeronomic Research, University of Alabama in Huntsville, AL, USA

Introduction

Modeling the solar wind and Coronal Mass Ejections (CMEs) in the inner heliosphere is critical for understanding these phenomenon and for space weather forecasting. Magnetohydrodynamic (MHD) codes based on classical spherical grids are limited by small cell sizes near the poles, leading to severe time step restrictions (See left panel of Fig. 1). In addition, the conventional MHD codes typically use 2^{nd} order accurate solvers. In this work, we present **HelioCubed**, a novel high-order MHD solver that overcomes these challenges using a **mapped cubed-sphere grid** and is designed to achieve **4^{th}-order accuracy in both space and time.** The code features **adaptive mesh refinement (AMR),** is GPU-portable, and supports boundary conditions from data-driven solar coronal models. This poster summarizes the core methodology, validation, and capabilities of HelioCubed, including applications to solar wind and CME simulations.

Numerical Method and Code Design

HelioCubed is developed using the **Proto C++ library**, a domain-specific language for structured-grid partial differential equation solvers that ensures performance portability across CPU and GPU systems. Proto is developed by the Applied Numerical Algorithms Group, Lawrence Berkeley National Laboratory. HelioCubed solves the ideal MHD equations using a mapped finite-volume formulation on cubed-sphere grids (McCorquodale & Colella 2011, Singh et al. 2025). We use **special quadrature techniques** to preserve radial symmetry and employ 4^{th} order **Runge-Kutta time stepping** to maintain high-order temporal accuracy. Fluxes are evaluated using high-order stencils, and non-oscillatory behavior near shocks is ensured with a 4^{th}-order limiter and high order linear and non-linear artificial viscosity. Divergence cleaning is implemented using the **Powell method** (Powell et al. 1999).

Validation and Performance

The order of convergence and accuracy of our method is tested using a radial expansion test with spherically symmetric initial density pulse with zero velocity. Fig. 2 shows the simulation result at a later time, indicating HelioCubed preserved radial flow to machine precision ($\sim 10^{-17}$). Convergence studies demonstrated an order of accuracy of **~4** with this test problem as shown in Table 1. We compared the time step stability of cubed-sphere and classical spherical grids. A simulation with 1 degree resolution on a spherical grid was stable with a time step of 2.8 s, while a cubed-sphere grid with same resolution can use a time step of 134.5 s, a **48× improvement**.

L^{∞} for $32\times16\times16$ (E1)	L^{∞} for $64\times32\times32$ (E2)	log(E1/E2)/log(2)
5.27258×10^{-6}	3.61363×10^{-7}	3.87

Table 1: Simulation Parameters and Results

Solar Wind and CME Simulations

HelioCubed supports solar wind modeling using boundary conditions (BCs) provided by either physics-based corona models or empirical models such as Wang-Sheeley-Arge (WSA). The BCs provided by corona models in HDF5 format are interpolated onto the cubed-sphere ghost cells using bilinear interpolation (Fig. 3). Simulations with 1 hour cadence BC reproduce key features such as **Parker spirals** and **fast-slow stream interactions** as seen in Fig 4. In this figure, we see that the 4^{th} order method produced much more structures SW, compared to the 2^{nd} order method. We also implemented the **constant-turn flux rope model** (Singh et al., 2022) to simulate CMEs with HelioCubed. Fig. 5 shows a test case of a CME with initial speed of 900 km/s. The left panel shows the initialized flux rope, and the right panel shows the expanding CME 1 day after initialization.

Adaptive Mesh Refinement (AMR)

AMR is implemented in HelioCubed using Proto's back-end support. Fig. 6 shows cell refinement in regions of high density near the equatorial current sheet. This feature enables efficient CME tracking and detailed resolution of solar wind structures without uniformly increasing computational cost.

Conclusions

HelioCubed represents a significant advancement in MHD modeling for heliophysics. Its use of mapped cubed-sphere grids removes time step limitations and improves accuracy. The code's GPU compatibility, high-order schemes, AMR capability, and integration with data-driven coronal boundary conditions make it a robust tool for inner heliosphere simulations and offers a foundation for next-generation space weather forecasting models.

References

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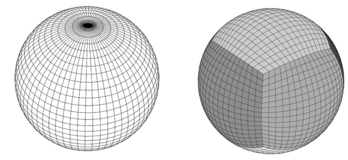

Fig. 1: (Left panel) Classical Spherical geometry. (Right Panel) Cubed sphere geometry, with faces colored in different shades of gray.

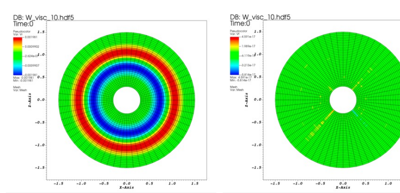

Fig. 2: Propagation of a spherically-symmetric acoustic pulse in cubed-sphere grid while preservation the radial flow symmetry, as shown by finite radial velocity (left panel) and zero (to machine accuracy) tangential velocity (right panel).

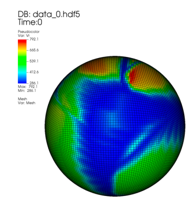

Fig. 3: The WSA boundary conditions interpolated on a cubed sphere grid of resolution 6 X 50 X 50.

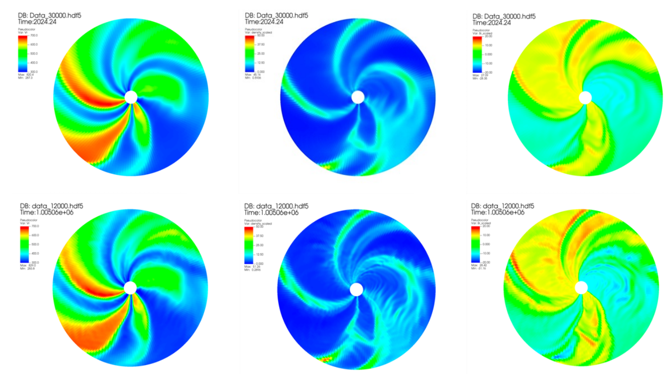

Fig. 4: Solar wind simulation results created using the WSA 1-hour cadence boundary conditions. The results are shown at about 2024-03-28 03:20 using 2^{nd} order method with spherical grid (top row) and 4^{th} order method with cubed-sphere grid (bottom row). The SW appears much more structured with the 4^{th} order simulation, while the 2^{nd} order simulation produces much more diffused results. Left column shows radial velocity, middle column shows density scaled with r^2 and the right column shows radial magnetic field scaled with r^2.

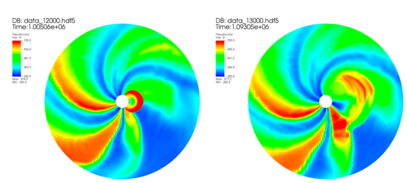

Fig. 5: An example simulation of a CME using constant turn flux rope using HelioCubed. The flux rope is inserted into the background solar wind at 2024-03-28 03:20 (left panel), which evolves as a CME as shown in the right panel at a later time of 2024-03-29 03:37.

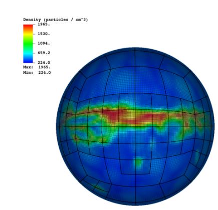

Fig. 6: The plasma density prescribed at the inner boundary of the cubed sphere domain at 21.5 Solar radii. Black lines represent the cell boundaries. AMR is used to obtain finer resolution near the equatorial current sheet.

The authors of this work are grateful for the support provided by jointly by the National Science Foundation (NSF) and NASA SWQU grant 2028154. TS is thankful for the support provided by the NASA grant 80NSSC25K7868.