Local Averaged Energy Transfer Rate in Plasma Turbulence

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Introduction

- Turbulence is a widespread phenomenon in space plasma, playing a crucial role in energy exchange and particle energization across various scales.
- A classical scenario of turbulence describes how energy is transferred from large energy-containing scales to small dissipation scales and is finally turned into heat, which is also called the energy cascade.
- The simple cascade picture was also postulated to apply to MHD turbulence and then validated with simulations and observation data.
- To obtain the local property of the energy transfer, we use different diagnostics in the inertial range and the dissipation range and conduct a comparison among different scales.

MHD Turbulence Model

- Incompressible MHD equations:

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{\nabla p}{\rho} + \frac{1}{\rho}\boldsymbol{j}\times\boldsymbol{B} + \nu\nabla^2\boldsymbol{u}$$

$$\partial_t \boldsymbol{B} - \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) = \eta\nabla^2\boldsymbol{B}$$

- After normalizing the MHD equations with respect to the Alfven time: $\tau_A = L/v_A$:

$$\frac{t}{\tau_A} := t,\quad \frac{x}{L} := x,\quad \frac{\boldsymbol{B}}{B_0} := \boldsymbol{b},\quad \frac{p}{\rho_0 v_A^2} := p$$

And introducing the Elsässer fields:

$$\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \boldsymbol{b}$$

We obtain

$$\partial_t \boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp}\cdot\nabla\boldsymbol{z}^{\pm} = -\nabla P + \nu^{+}\nabla^2\boldsymbol{z}^{\pm} + \nu^{-}\nabla^2\boldsymbol{z}^{\mp}$$

Where $P = p + \frac{1}{2}b^2$, $\nu^{\pm} = \frac{1}{2}(\nu \pm \eta)$

- The energy transfer channels among different scales and forms:

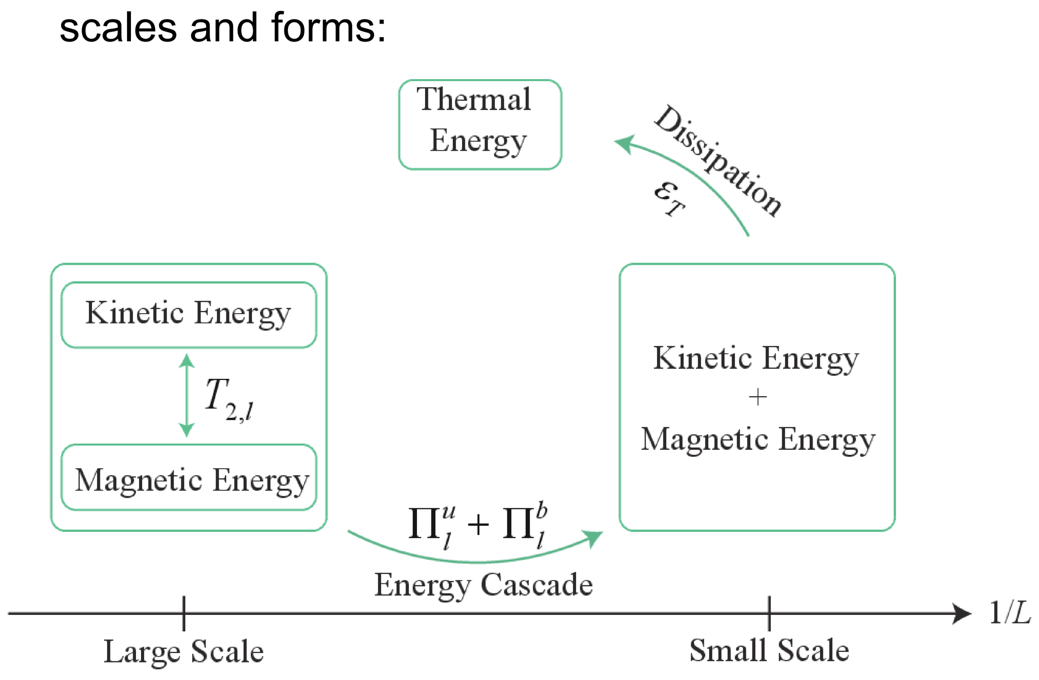

Fig. 1. (Adapted from Y. Yang 2019 *Energy Transfer and Dissipation in Plasma Turbulence*, *Springer* P74)

- In Fig. 1, the dissipation rate ε is the dissipation rate of the Elsasser fields

$$\varepsilon = \frac{1}{2}(\varepsilon^{+} + \varepsilon^{-})$$

$$\varepsilon^{\pm} = \frac{1}{2}\partial_t(\boldsymbol{z}^{\pm})^2 = \nu^{+}S_{ij}^{\pm}S_{ij}^{\pm} + \nu^{-}S_{ij}^{\pm}S_{ij}^{\mp}$$

where

$$S_{ij}^{\pm} = \frac{1}{2}(\partial_i z_j^{\pm} + \partial_j z_i^{\pm})$$

- $\Pi_l^u = -(\bar{\tau}_l^u + \bar{\tau}_l^b) : \nabla\bar{\tau}_l$, is the kinetic energy flux across scale l due to the subscale stresses.
- $\Pi_l^b = -\bar{\boldsymbol{\mathcal{E}}}_l\cdot(\nabla\times\bar{\boldsymbol{B}}_l)$, is the magnetic energy flux across scale due to the subscale electromotive force.
- $T_{2,l} = \bar{\boldsymbol{B}}_l\cdot\nabla\bar{\boldsymbol{u}}_l\cdot\bar{\boldsymbol{B}}_l$, is the exchange of large-scale kinetic and magnetic energy due to large-scale magnetic line stretching.
- Here we apply a low-pass filter $\bar{\boldsymbol{a}}_l(\boldsymbol{x},\boldsymbol{v},t) = \int \boldsymbol{a}(\boldsymbol{x}+\boldsymbol{r},\boldsymbol{v},t)G_l(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$, $G_l(\boldsymbol{r}) = l^{-3}G(\boldsymbol{r}/l)$, $G(\boldsymbol{r})$ is a normalized boxcar window function.

MHD Simulation Results

- We perform a driven 3D MHD simulation (Jiang et al. 2023) with a resolution of 1024^3. The grid size is $\Delta = 2\pi/1024$ No guide magnetic field is applied. Viscosity ν equals resistivity μ for each simulation. The Kolmogorov scale is $\eta \approx 0.8\Delta$.

Resolution(3D)	1024
$E_u = E_b$	0.5
k range	1~5
$\nu = \mu$	1.0×10^{-3}
Re_λ	290

- To reveal the local characteristic of the dissipation rate, we calculate the locally averaged dissipation rate ε_r under different lag r, $\varepsilon_r = \frac{3}{4\pi r^3}\int_{|\boldsymbol{x}'|\le r}\varepsilon\,(\boldsymbol{x}+\boldsymbol{x}')\,\mathrm{d}\boldsymbol{x}'$.

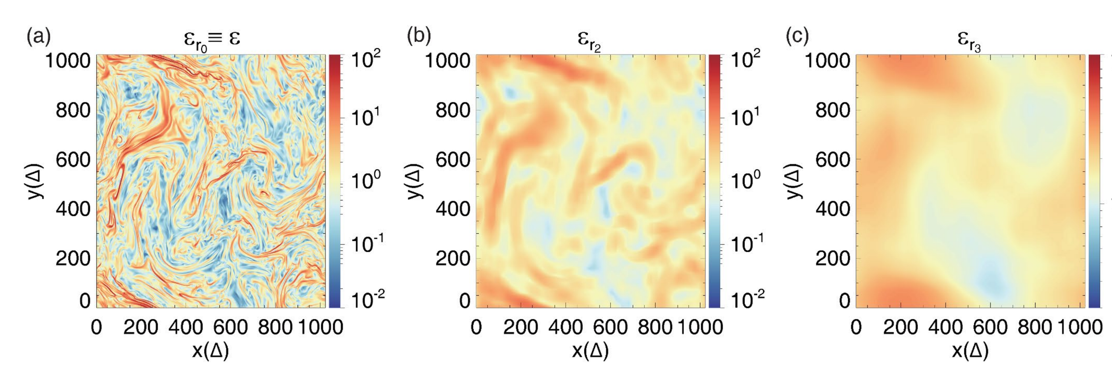

Fig. 2. Locally averaged dissipation rate at (a) $r_0 = 0$, (b) $r_2 = 46\Delta$ (inertial range), and (c) $r_3 = 245\Delta \sim L_f$ (energy-containing range).

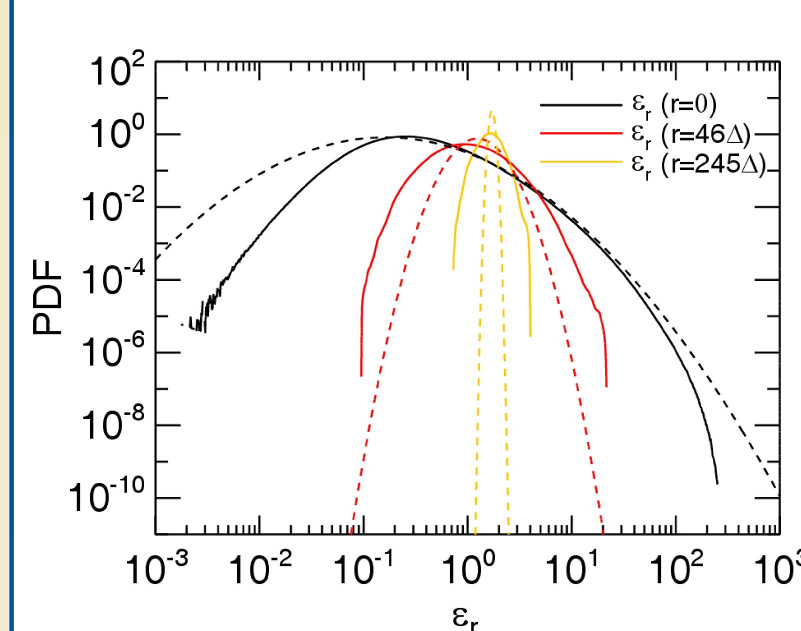

Fig. 3. PDFs of locally averaged dissipation rate ε_r at $r_0 = 0$, $r = 46\Delta$, $r = 245\Delta$.

- Local energy transfer (LET) rate: (Sorriso-Valvo, L. *et al*, 2018, *J. Plasma Phys*. vol. **84**, 725840201)
  - $\xi_r = \frac{1}{2}(\xi_r^{+} + \xi_r^{-})$, $\xi_r^{\pm} = -\frac{d}{4}\,\delta z_{\parallel}^{\mp}\delta z_i^{\pm}\delta z_i^{\pm}$

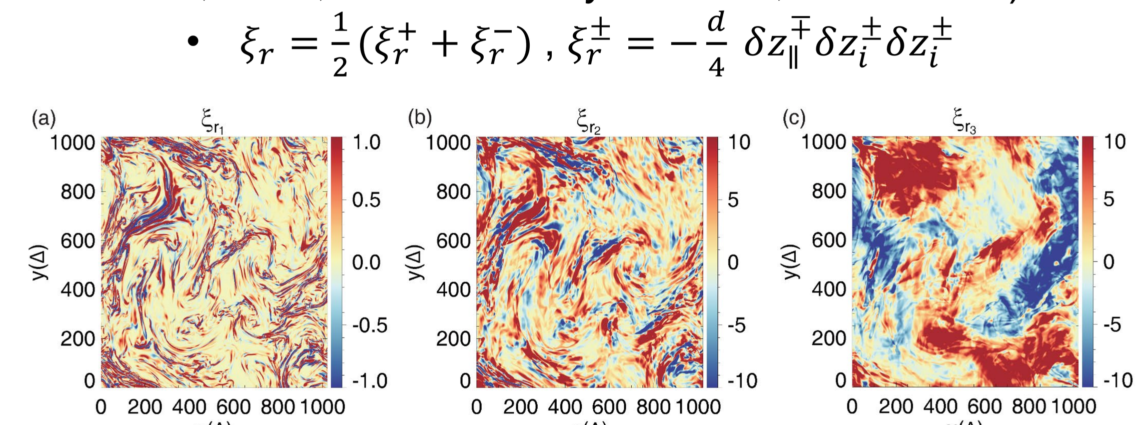

Fig. 4. Local energy transfer rate at the scale (a) dissipation range $r_1 = 3\Delta$, (b) inertial range r_2, and (c) energy-containing range r_3.

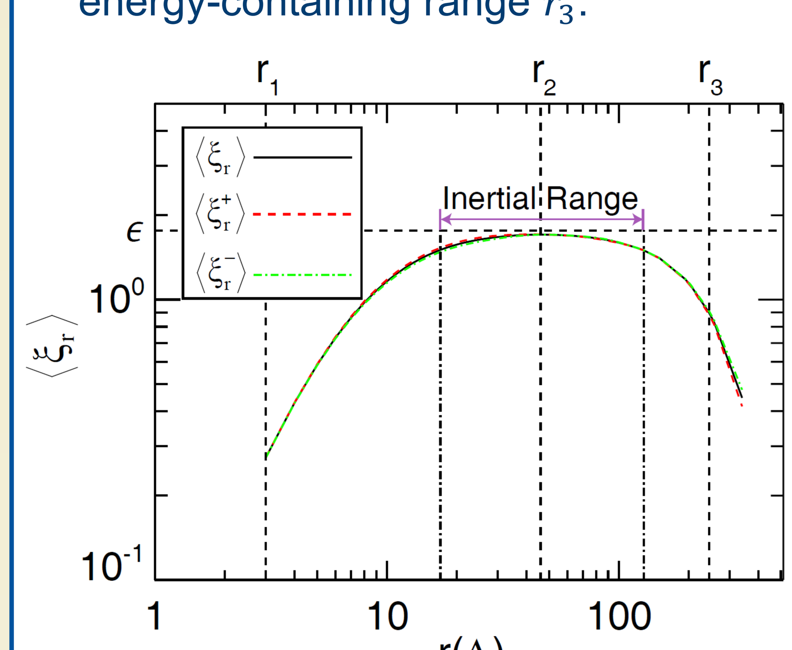

Fig. 5. Third-order structure functions $\langle\xi_r\rangle$ and $\langle\xi_r^{\pm}\rangle$ vs separation length r. Inertial range is $17 \sim 128\Delta$

- Scale-filtered energy flux: total energy flux across scale r, $\Pi_r = \Pi_r^u + \Pi_r^b$

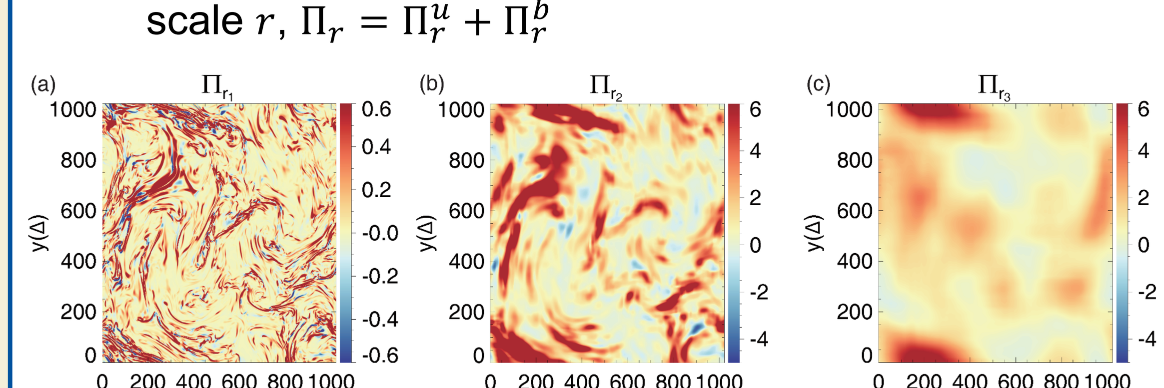

Fig. 6. Total energy flux across scales in (a) dissipation range r_1, (b) inertial range r_2, and (c) energy-containing range r_3.

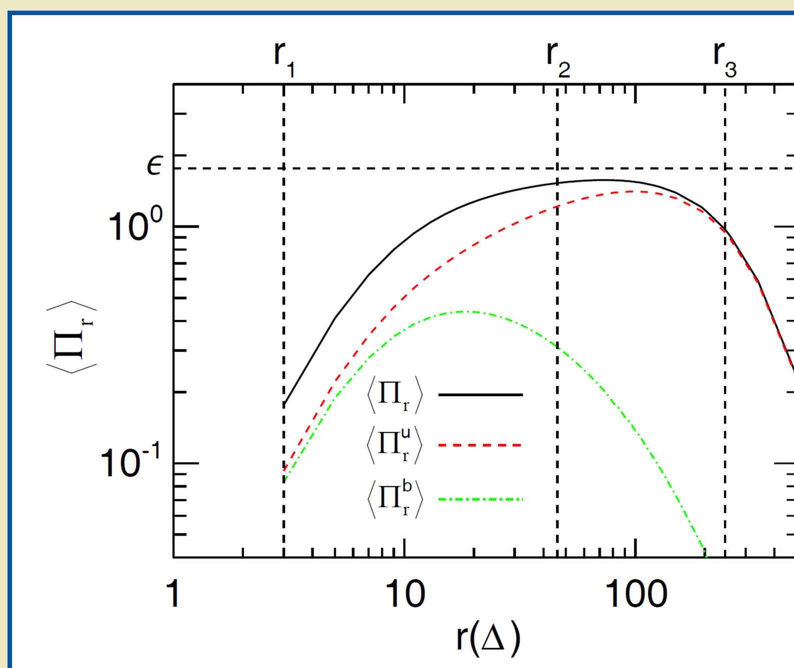

Fig. 7. Spatial averaged $\Pi_r^{u,b}$ and Π_r vs scale r

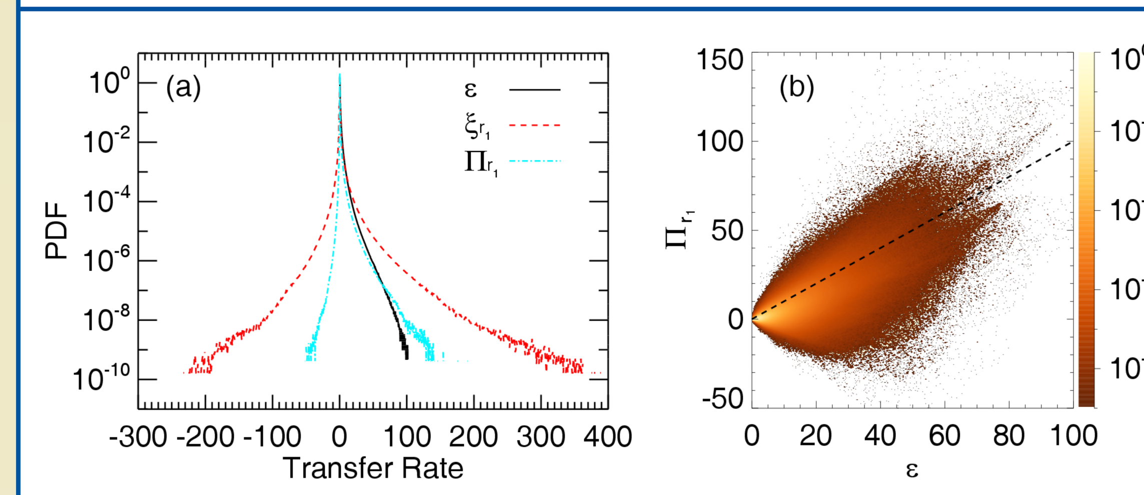

Fig. 8. (a) PDFs of dissipation rate ε, LET ξ_{r_1} , and the scale-filtered flux Π_{r_1}. (b) Joint PDF of dissipation rate ε and the scale-filtered flux Π_{r_1}.

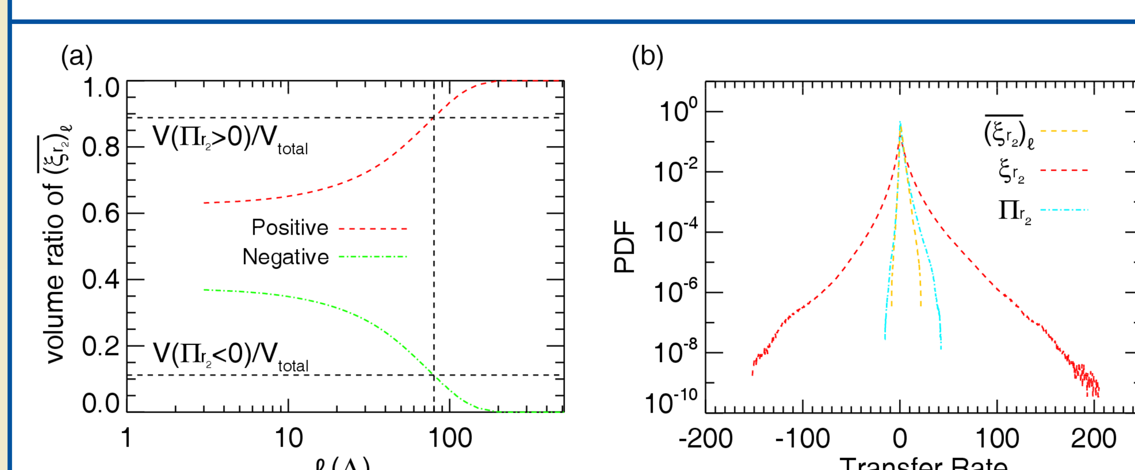

Figure 9. (a) Volume ratios of the locally averaged $\overline{(\xi_{r_2})}_\ell$ over size ℓ. (b) PDFs of $\overline{(\xi_{r_2})}_\ell$ at $\ell = 80\Delta$, LET ξ_{r_2} , and the scale-filtered flux Π_{r_2}.

- Cross correlations between energy transfer measurements ε, ξ_{r_2}, and Π_{r_2}

$$R(f,g,r) = \frac{\langle(f(\boldsymbol{x}+\boldsymbol{r}) - \langle f\rangle)(g(\boldsymbol{x}) - \langle g\rangle)\rangle}{\langle(f(\boldsymbol{x}) - \langle f\rangle)(g(\boldsymbol{x}) - \langle g\rangle)\rangle}$$

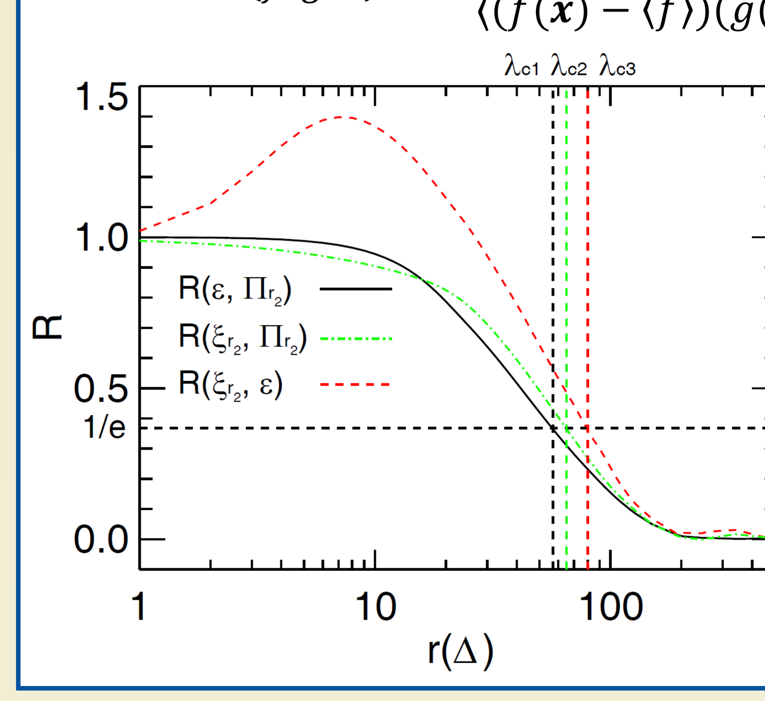

Fig. 10. Correlation functions vs scale r

Directions for Future Research

To generalize our research into kinetic system, we will perform PIC simulations, where the dissipation ε is unavailable. Instead, we will calculate pressure–strain interaction $(\vec{\boldsymbol{P}}\cdot\nabla)\cdot\mathbf{u}$.

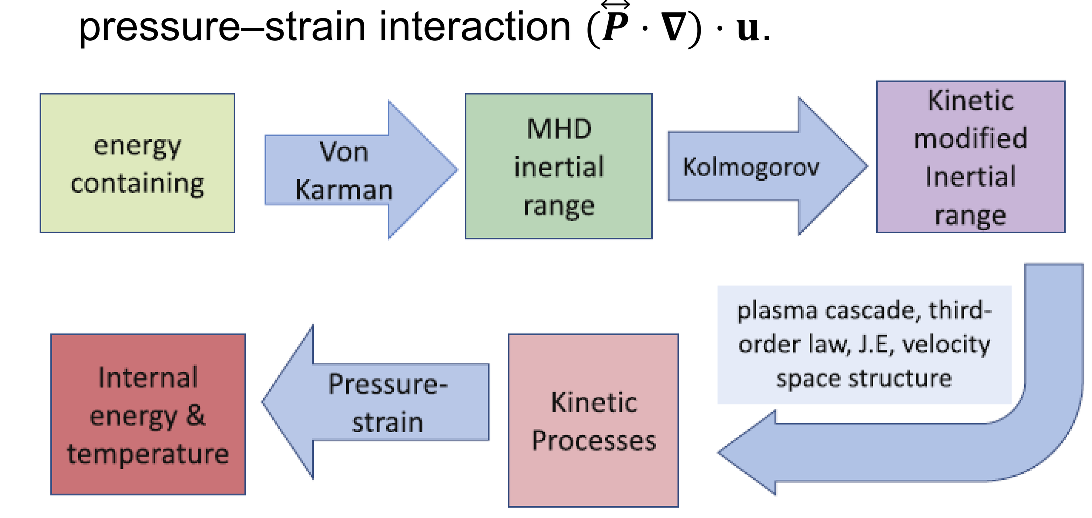

Fig. 11. (Adapted from William H. Matthaeus *et al*, 2020 *ApJ* **891** 101)

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